SUPPLEMENTAL MATERIALS PROJECT DIRECTORS

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About these materials

This supplement to *Integrated Mathematics*: *A Modeling Approach Using Technology* was created in response to the requests of teachers who use SIMMS IM materials. Some asked for extra problems to help students solidify their knowledge of the mathematics skills in the curriculum. Some wanted more problems that illustrate applications of mathematical content. Others desired additional assessment tools that could be used in conjunction with those already included in each module.

The materials in this copy of *Additional Review and Periodic Assessment* have been written specifically for Level 2. The review pages offer a selection of straightforward skill problems, along with one or two context problems, for each activity in each of the 15 modules. The periodic assessments each consist of four or five problems and are intended to be completed in less than one class period.

A principal feature of the SIMMS IM curriculum is the depth and breadth of the mathematics covered. All modules are designed to help students understand concepts and learn the specific skills necessary to become effective problem-solvers. The problems included here are not intended to replace the work students do in Explorations, Discussions, and Assignments. They can be used, however, to provide extra practice for certain students, groups of students, or entire classes.

Thank you for selecting the SIMMS IM curriculum for your school—and for your commitment to making a positive impact on the mathematics education of each student. Thanks also to the fine teachers who helped prepare these materials: Clay Burkett, Terri Dahl, Peggy Lynn, Sue Moore, Mike Trudnowski, and Lisa Wood.

—Terry Souhrada

Supplementary Materials Project Coordinator

Level 2: Volume 1—Marvelous Matrices

Additional Review—for use with Activity 1

1.1. Identify the dimensions of each of the following matrices:

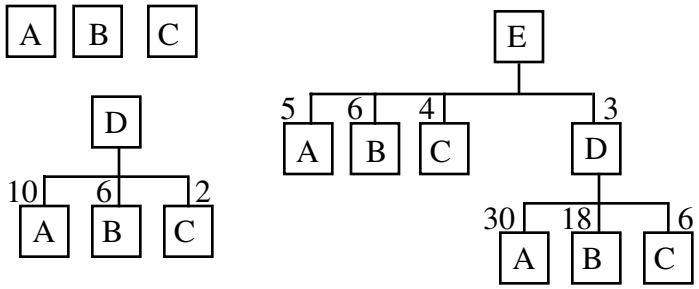
a. $\begin{bmatrix} 3 & 5 \end{bmatrix}$ **b.** $\begin{bmatrix} 3 & 6 \\ -1 & 11 \end{bmatrix}$ **c.** $\begin{bmatrix} 4 & 2 & 16 & 7 \\ 3 & 1 & 2 & -9 \end{bmatrix}$

d. $\begin{bmatrix} -3 & 1.8 & 1/3 \\ 8 & 0 & 0 \\ -1 & 2 & 5 \end{bmatrix}$ **e.** $\begin{bmatrix} 4 & 5 \\ 3 & 1 \\ 2 & 0 \end{bmatrix}$ **f.** $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

1.2. Enter the numbers –5, –1, 0, 2, 3, 7 in a matrix with dimensions 3×2.

1.3. How many elements are there in a matrix with dimensions 5×3?

1.4. The diagram below is a requirement graph.



a. Identify the simple components from the requirement graph.

b. Create a requirement matrix for this graph.

c. How many of each simple component are needed to fill 10 orders of item E?

1.5. A do-it-yourself furniture company ships the accompanying hardware in two different types of packages: plastic bags and cardboard cartons. Each plastic bag contains 3 bolts and 6 nuts. Each cardboard carton contains 2 of the bags, along with some additional bolts and nuts. The requirement matrix for these items is shown below.

$$\mathbf{S} = \begin{array}{c} \\ \text{B} \\ \text{N} \\ \text{P} \\ \text{C} \end{array} \begin{array}{c} \begin{array}{cccc} \text{B} & \text{N} & \text{P} & \text{C} \end{array} \\ \begin{bmatrix} 1 & 0 & 3 & 7 \\ 0 & 1 & 6 & 16 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

a. Create a requirement graph for this situation.

b. Identify the simple components of matrix **S**.

c. How many of each simple component are needed to fill a 10-carton order?

Additional Review—for use with Activity 2

2.1. For each matrix expression below, perform the indicated operation, if possible. If the operation is not possible, explain why.

a. $\begin{bmatrix} 4 & 5 \\ 3 & 1 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 8 & 4 \\ -1 & -2 \\ 0 & 6 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 \\ -3 & 0 \\ 1/3 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ -1 & 11 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 1 \\ 2 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$

d. $\begin{bmatrix} r & 2s \\ p & -q \end{bmatrix} + \begin{bmatrix} r & s \\ m & n \end{bmatrix}$

2.2. Perform the indicated operation, if possible, for each of the following. If the operation is not possible, explain why.

a. $-3 \bullet \begin{bmatrix} 8 & 4 \\ -1 & -2 \\ 0 & 6 \end{bmatrix}$

b. $\frac{1}{2} \bullet \begin{bmatrix} 6 & 16 \\ -4 & 11 \end{bmatrix}$

c. $2 \bullet \begin{bmatrix} 4 & 2 & 16 & 7 \\ 3 & 1 & 2 & -9 \end{bmatrix}$

d. $a \bullet \begin{bmatrix} r & s \\ m & n \end{bmatrix}$

2.3. For each matrix expression below, perform the indicated operations, if possible. If an operation is not possible, explain why.

a. $\begin{bmatrix} 3 & 5 \\ -4 & 11 \end{bmatrix} + 2 \bullet \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

b. $\frac{1}{2} \bullet \begin{bmatrix} 8 & 4 \\ 2 & -2 \\ 0 & 6 \end{bmatrix} + 3 \bullet \begin{bmatrix} 2 & 2/3 \\ 5 & 1 \\ -3 & 0 \end{bmatrix}$

2.4. An automobile dealership stocks three different types of cars: compacts (C), sedans (S), and luxury cars (L). Each of these types comes in four different versions: two-door (2D), four-door (4D), hatchback (HB), and station wagon (SW). The following matrix shows the dealership's current inventory:

$$\mathbf{I} = \begin{array}{c} \\ \text{C} \\ \text{S} \\ \text{L} \end{array} \begin{array}{c} \begin{array}{cccc} \text{4D} & \text{2D} & \text{HB} & \text{SW} \end{array} \\ \begin{bmatrix} 3 & 10 & 5 & 1 \\ 15 & 9 & 3 & 4 \\ 7 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

a. Identify the dimensions of matrix **I**.

b. The dealership wishes to double its current inventory for the upcoming year. Write a matrix that represents their new inventory.

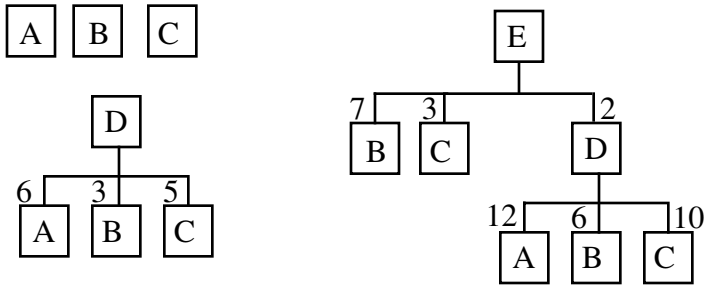
c. How many sedans will be included in the new inventory?

Periodic Assessment 1—for use after Activity 2

1. Identify the dimensions of each of the following matrices:

a. $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 3 & 0 \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & 25 \\ 20 & 8 \\ 13 & 19 \\ 45 & 22 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. Create a requirement matrix for the requirement graph shown below.



3. Perform the indicated operation, if possible, for each of the following. If the operation is not possible, explain why.

a. $\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 3 \\ 1 & 0 \\ 5 & -2 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 & 3 \\ -5 & 17 \end{bmatrix} - \begin{bmatrix} -3 & 6 \\ -1 & 11 \end{bmatrix}$

c. $\begin{bmatrix} 5 & -3 & 7 \\ -3 & 1 & 6 \end{bmatrix} - \begin{bmatrix} -4 & -2 & 8 \\ 6 & 0 & 3 \end{bmatrix}$ **d.** $\begin{bmatrix} 2 & -3 & 0 \\ -2 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 7 & 3 \\ 1 & 0 \end{bmatrix}$

Additional Review—for use with Activity 3

3.1. For each matrix expression below, perform the indicated operation, if possible. If the operation is not possible, explain why.

a. $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 5 & -1 \end{bmatrix} \bullet \begin{bmatrix} 8 & 4 \\ -1 & -2 \\ 0 & 6 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 4 & -1 \\ 1 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 3 & 4 \\ 2 & 6 \\ -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 2 & 3 & 1 \\ 2 & 5 & -3 \end{bmatrix}$

d. $\begin{bmatrix} -2 & -3 & 4 \\ 1 & 3 & 9 \end{bmatrix} \bullet \begin{bmatrix} 4 & 6 \\ -2 & -1 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \bullet \begin{bmatrix} 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

f. $\begin{bmatrix} r & 2r \\ p & -q \end{bmatrix} \bullet \begin{bmatrix} r & s \\ r & 3s \end{bmatrix}$

3.2. If matrix **L** has dimensions $p \times q$ and matrix **N** has dimensions $s \times t$, what must be true so that $\mathbf{L} \bullet \mathbf{N}$ is possible? What are the dimensions of $\mathbf{L} \bullet \mathbf{N}$?

3.3. A clothing store sells three types of shoes: athletic, dress, and casual. Each type of shoe comes in three styles: slip-on, buckle, and tie. The following matrix shows the store's inventory:

$$\mathbf{I} = \text{type} \begin{matrix} \\ \text{A} \\ \text{D} \\ \text{C} \end{matrix} \begin{matrix} \text{style} \\ \text{S} \quad \text{B} \quad \text{T} \end{matrix} \begin{bmatrix} 1 & 6 & 35 \\ 10 & 2 & 9 \\ 12 & 7 & 2 \end{bmatrix}$$

The average wholesale cost of each pair of athletic shoes is \$21, of dress shoes is \$30, and of casual shoes is \$23. Use matrix methods to calculate the total value of all shoes in stock.

Periodic Assessment 2—for use after Activity 3

1. For each matrix expression below, perform the indicated operation, if possible. If the operation is not possible, explain why.

a. $2 \bullet \begin{bmatrix} -2 & -3 & 4 \\ 3/4 & 0.6 & 0 \end{bmatrix}$

b. $\frac{1}{3} \bullet \begin{bmatrix} 6 & 15 \\ -4 & 11 \end{bmatrix}$

2. Perform the indicated operation, if possible, for each of the following. If the operation is not possible, explain why.

a. $\begin{bmatrix} 7 & 3 \\ -2 & -2 \\ 0 & 5 \end{bmatrix} \bullet \begin{bmatrix} -2 & 0 & 1 \\ 3 & 1/2 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \bullet \begin{bmatrix} 4 & -1 & 0 \\ 1 & -3 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} \bullet \begin{bmatrix} -2 & 1 & 6 \\ -8 & 0 & 3 \\ 2 & -1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} -2 & 1 & 6 \\ -8 & 0 & 3 \\ 2 & -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}$

3. Is matrix multiplication commutative? Justify your response.

4. A sporting goods store sells equipment packages for both downhill (DH) and cross country (CC) skiing. For each of these styles, packages are available for three different skill levels: advanced (A), intermediate (I), and beginner (B). Matrix **S** shows the number of each kind of ski package in stock. Matrix **P** indicates the price of each package.

$$\mathbf{S} = \begin{array}{c} \\ \text{CC} \\ \text{DH} \end{array} \begin{array}{c} \begin{array}{ccc} \text{B} & \text{I} & \text{A} \end{array} \\ \begin{bmatrix} 15 & 25 & 10 \\ 25 & 45 & 10 \end{bmatrix} \end{array}$$

$$\mathbf{P} = \begin{array}{c} \\ \text{B} \\ \text{I} \\ \text{A} \end{array} \begin{array}{c} \begin{array}{cc} \text{CC} & \text{DH} \end{array} \\ \begin{bmatrix} \$69.95 & \$79.95 \\ \$274.95 & \$324.95 \\ \$349.95 & \$474.95 \end{bmatrix} \end{array}$$

a. If the store sells all of its stock, what will be the total sales of cross country packages? of downhill packages? (Hint: Find the product **S** • **P**.)

b. If the store sells all of its stock, what will be the total sales of beginner packages? of intermediate packages? of advanced packages? (Hint: Find the product **P** • **S**.)

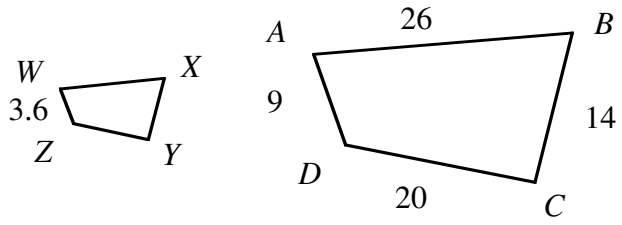
Level 2: Volume 1
A New Angle on an Old Pyramid

Additional Review—for use with Activity 1

1.1 Explain what it means for two geometric figures to be similar.

1.2 The ratio of the lengths of two corresponding sides of two similar polygons is called the _____.

1.3 Complete Parts **a–d** below given that $ABCD \sim WXYZ$.



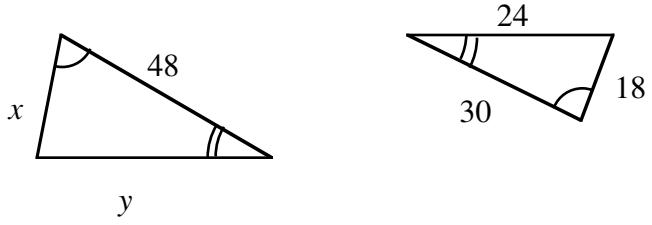
a. What is the scale factor of *ABCD* to *WXYZ*?

b. What are the lengths of $\overline{WX}$, $\overline{XY}$, and $\overline{YZ}$?

c. What are perimeters of *ABCD* and *WXYZ*?

d. What is the ratio of the perimeters of these quadrilaterals?

1.4 Use the diagram below to determine the values of x and y.



1.5 Hans has a dead tree in his backyard that must be removed. He is considering cutting it down himself, but worries that it might hit the house. The distance from the tree to the house is 18 m. Hans, who is 198 cm tall, casts a shadow 132 cm long. The tree's shadow is 9.6 m long. Is the tree tall enough to hit the house as it falls? Justify your response.

Additional Review—for use with Activity 2

2.1 Determine whether each of the following sets of numbers could be the side lengths of a triangle. If so, identify the triangle as right, acute, or obtuse. Justify your responses.

a. 9, 15, 12

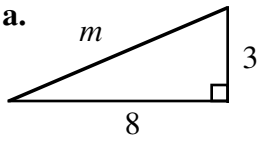
b. 3, 5, 9

c. 8, 8, 8

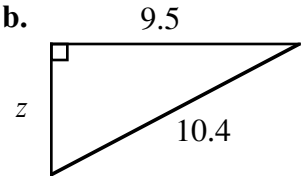
d. 14, 11, 23

2.2 Determine the unidentified side length in each of the right triangles below.

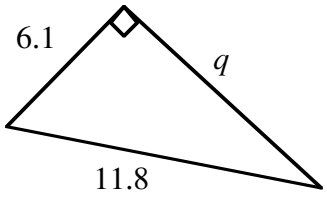
a.



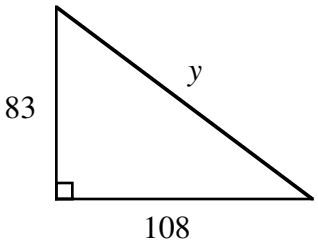
b.



c.



d.



2.3 Write the converse of the statement "If the diagonals of a quadrilateral are perpendicular, then the quadrilateral is a rhombus."

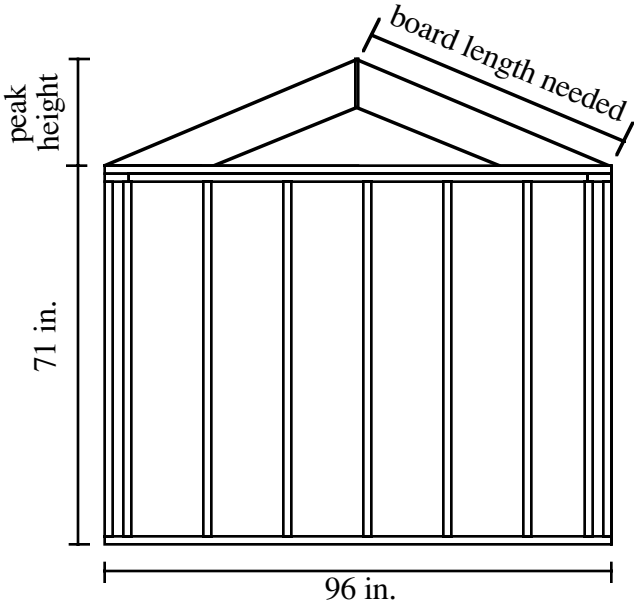
2.4 Find the distance between each of the following pairs of points.

a. (9,14) and (4,1)

b. (–12,8) and (7,–4)

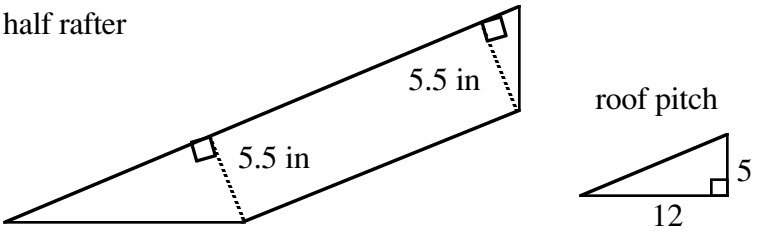
c. (–3,–6) and (–9,8)

2.5 An industrial arts class wants to build storage sheds for sale. The diagram below shows the unfinished design for an end wall. The roof has a pitch of $5/12$. This means that for every 12 inches of horizontal length, the roof rises 5 inches vertically. The roof's peak is the highest point on the shed.



a. The door to the school shop is 9 feet high and 9 feet wide. If the sheds are built inside the shop, will the students be able to be move them out the door?

b. The boards that support the roof are rafters. A half rafter is shown below.



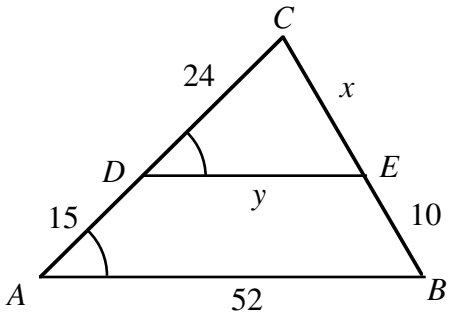
The lumber used for the rafters is $1\frac{1}{2}$ inches thick and $5\frac{1}{2}$ wide. A board with these dimensions is called a "two-by-six." Determine the length of the two-by-six needed to create a half rafter. (Hint: The two right triangles marked on the half rafter in the diagram are similar to the roof pitch.)

c. Each shed requires 6 rafters. As shown in the following table, two-by-six lumber is sold in several different lengths. Determine the minimum cost for the lumber needed to make 6 rafters.

Length (in feet)	**Cost**
8	\$3.75
10	\$4.69
12	\$5.63
14	\$7.12
16	\$8.51

Periodic Assessment 1—for use after Activity 2

1. Determine the values of x and y in the diagram below.



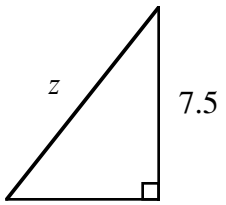
2. Determine whether each of the following sets of numbers could be the side lengths of a triangle. If so, identify the triangle as right, acute, or obtuse.

a. 15, 31, 14

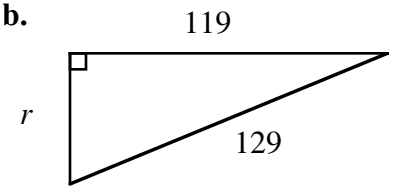
b. 21, 16, 17

3. Determine the unidentified side length in each of the right triangles below.

a.



b.



4. Calculate the distance between the coordinate points (6,–10) and (14,19).

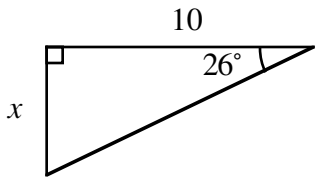
5. The extension ladder on a fire engine is 30 m long. The platform where the base of the ladder rests on the truck is 2.5 m above the pavement. If the engine can park no closer than 6 m to a burning building, could a firefighter reach a window on the 10th floor? (Assume that each floor is approximately 3 m high.)

Additional Review—for use with Activity 3

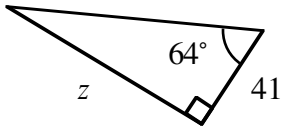
3.1 Write a definition of the term *dihedral angle*.

3.2 Determine the value of the unidentified measure in each triangle below.

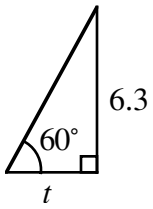
a.



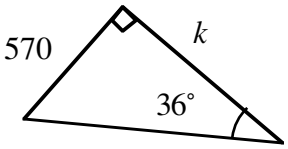
b.



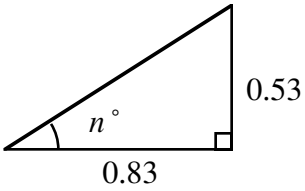
c.



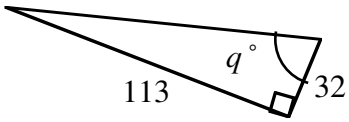
d.



e.



f.



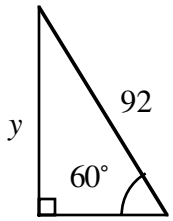
3.3 To estimate a tree's height, a landscape architect paced off a distance 15 m from the trunk. From that point, she used a clinometer to measure the angle of elevation to the top of the tree: 72°. The landscape architect is 1.68 m tall. What is the height of the tree?

Additional Review—for use with Activity 4

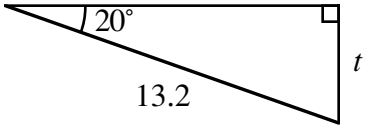
4.1 Draw a right triangle, labeling all sides and vertices. Choose one acute angle and write the formulas for the three trigonometric ratios relative to that angle.

4.2 Determine the value of the unidentified measure in each triangle below.

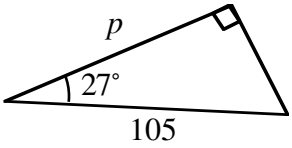
a.



b.



c.



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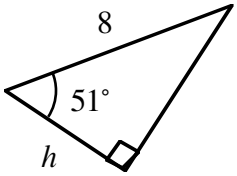
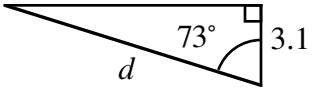
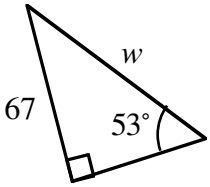


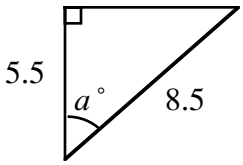
e.



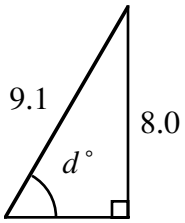
f.



g.



h.

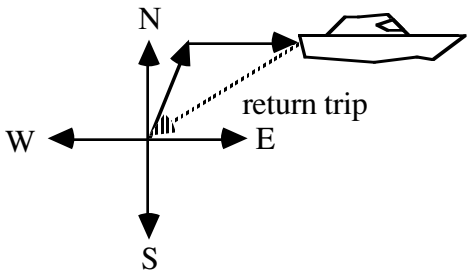


4.3 Find the height of an isosceles triangle with a vertex angle of 48° and congruent legs with a length of 5.8 cm.

4.4 A ship left its home port and sailed 85 km in a direction 29° east of north, then turned and headed due east for 90 km.

a. How far is the ship from it home port?.

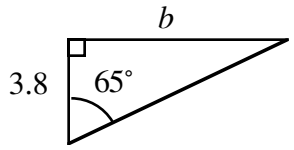
b. In what direction should the ship travel to return directly home?



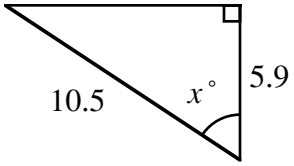
Periodic Assessment—for use after Activity 4

1. Determine the unidentified measure in each triangle below.

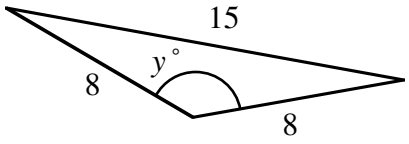
a.



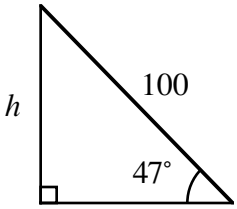
b.



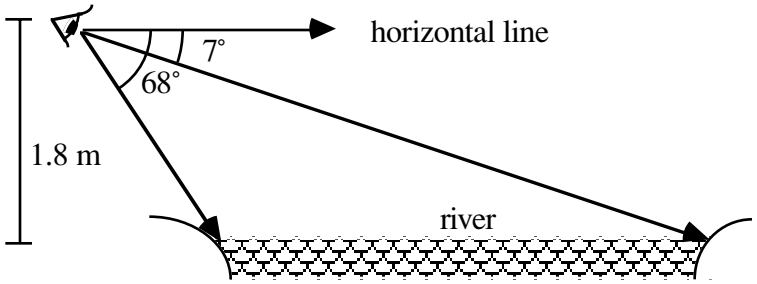
c.



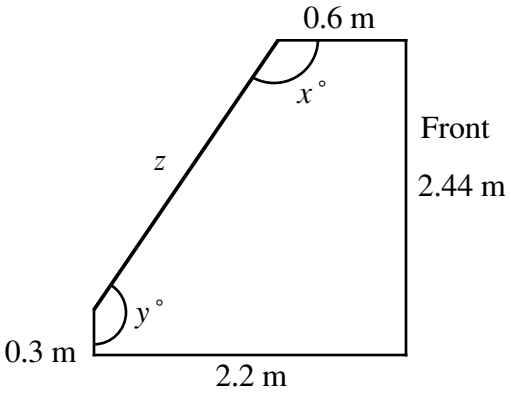
d.



2. Randy wants to estimate the width of the river that flows behind his house. Using a clinometer, he measures the angle of depression (the angle measured from a horizontal line) from his eye to each bank of the river. Considering that his eye is 1.8 m above the ground, find the width of the river.



3. Belle is building a frame for a soccer goal. Her coach gave her the diagram below, which includes the lengths for the vertical and horizontal supports. Determine the rest of the measurements needed to build the frame.



Level 2: Volume 1
When to Deviate from a Mean Task

Additional Review—for use with Activity 1

1.1 The student council at Carson School includes 15 freshmen, 18 sophomores, 22 juniors and 25 seniors. Create a histogram of this information.

1.2 To raise funds, the debate team at Carson School sells bagels. Last week they sold a total of 180 bagels—80 plain, 45 blueberry, 30 whole wheat, and 25 parmesan. Describe how you would display this information in a pie chart, including the measure of the central angle of each sector.

1.3 Carson High School draws its students from three different middle schools. The current enrollment of 1500 students includes 65% from Emerson Middle School, 25% from Longfellow Middle School, and 10% from Alcott Middle School. Determine the number of students from each middle school currently enrolled at Carson. Describe how you would display this information in a pie chart, including the measure of the central angle of each sector.

1.4 The students in a math class rolled a die 94 times, obtaining the following results: 13 ones, 16 twos, 18 threes, 12 fours, 20 fives, and 15 sixes.

a. Create a frequency table to organize this data.

b. Draw a histogram of the data.

c. Draw a pie chart of the data.

Additional Review—for use with Activity 2

2.1 Find the mean of each of the following sets of numbers.

a. 7, 3, 5, 5, 4, 6

b. 23, 53, 45, 17

c. 9.3, 2.6, 4.4, 8.1, 5.9

2.2 Find the median of each set of numbers below.

a. 4, 7, 9, 2, 2, 3, 5, 1

b. 13.1, 15.2, 14.6, 13.35, 15.5, 14.16, 14.32, 15.07, 13.05

c. 35, 72, 69, 44, 29, 14, 16, 83, 67, 88

2.3 Find the mode of each of the following sets of numbers.

a. 1, 5, 3, 2, 3, 4, 5, 6, 5

b. 68, 84, 75, 84, 95, 76, 75, 82, 97

c. 4.3, 5.7, 6.2, 4.7, 5.5, 6.7, 5.2, 6.5

2.4 Find the range of each set of numbers below.

a. 37.6, 55.3, 42.8, 61.1, 28.6, 84.9, 69.2

b. 15, 42, 27, 13, 41, 35, 38, 55, 14

c. 1.1, 1.5, 1.42, 1.06, 1.54, 1.32, 1.83, 1.8, 1.7, 1.73, 1.54

2.5 A class of students received the following scores on a quiz: 83, 66, 52, 75, 73, 71, 84, 84, 71, 83, 89, 94, 62, 97, 89, 75, 58, 91, 75, 88. Construct a stem-and-leaf plot of these scores. Include a legend.

Additional Review—for use after Activity 3

3.1 Find the median, lower quartile, upper quartile, and interquartile range of each of the following sets of numbers.

a. 23, 56, 76, 41, 54, 22, 72, 55, 46, 76

b. 3.5, 2.2, 4.9, 3.6, 2.8, 4.5, 3.7, 3.9, 4.6

3.2 Describe the values of any outliers in each data set below.

a. a data set with a lower quartile of 11 and an upper quartile of 18

b. a data set with a lower quartile of 27.5 and an upper quartile of 44.3

3.3 Create a box-and-whisker plot for each of the following sets of data.

a. 45, 34, 16, 42, 36, 47, 49, 33, 50, 38

b. 88, 90, 82, 97, 84, 15, 83, 94, 52, 115, 84, 82, 65,100, 82.

c. 3.5, 4.2, 3.7, 2.2, 4.5, 5.1, 4.4, 3.6, 2.1, 5.9, 6.2, 0.4

3.4 The table below shows the lowest recorded temperature for each month in Juneau, Alaska. Create a box plot for this information.

Month	**Temperature (°F)**
January	–22
February	–22
March	–15
April	6
May	25
June	31
July	36
August	27
September	23
October	11
November	–5
December	–21

Source: *Statistical Abstract of the United States*. Washington, DC: U.S. Bureau of the Census, 1995. p. 244.

Periodic Assessment—for use after Activity 3

1. In generating 20 random numbers, the following results occurred: 3 ones, 5 twos, 7 threes, 2 fours, and 3 fives.

a. Organize these results using a frequency table.

b. Create a histogram of the data.

c. Create a pie chart of the data.

d. Find the mean, median, mode, and range of the data.

2. As part of their physical fitness program, 20 students ran a mile. The following table contains their times, to the nearest 0.1 min. Use this information to make a back-to-back stem-and-leaf plot that compares the times for boys and girls.

Boys	8.1	7.5	6.1	6.5	8.6	9.6	8.1	4.3	12.2	
Girls	13.5	8.9	8.5	11.2	9.5	7.2	7.5	6.6	5.9	9.9	11.4

3. Create a box-and-whisker plot for the following set of numbers: 24, 36, 56, 8, 45, 29, 100, 30, 1, 60, 42, and 35.

Additional Review—for use with Activity 4

4.1 Evaluate each of the following expressions.

a. $|5|$ **b.** $|-6|$ **c.** $|3-7|$

d. $5|-2|$ **e.** $|-4\cdot -7|$ **f.** $|-35|\cdot|20|$

4.2 Evaluate each expression below under the given conditions.

a. $|a|$, if $a > 0$ **b.** $|y|$, if $y < 0$

c. $|x+y|$, if $x+y > 0$ **d.** $|a-b|$, if $a-b < 0$

e. $-4|-2y|$, if $-2y < 0$ **f.** $\left|x^2+y^2\right|$

4.3 Calculate the mean absolute deviation for each set of numbers below.

a. 87, 94, 72, 65, 97, 77

b. 34.6, 54.6, 42.8, 51.5, 60.9, 22.7, 43.9

4.4 Last week at Carson School, one teacher recorded the following numbers of absences: 4 on Monday, 2 on Tuesday, 2 on Wednesday, 3 on Thursday, and 6 on Friday.

a. Create a histogram of the data and draw a horizontal line that indicates the mean.

b. Calculate the mean absolute deviation for this data.

Additional Review—for use with Activity 5

5.1 Simplify each of the following expressions.

a. $\sqrt{25}$ **b.** $\sqrt{3/4}$ **c.** $\sqrt{-5}$

d. $\sqrt{\dfrac{3+5+8}{7}}$ **e.** $\sqrt{\dfrac{(3-7)^2}{8}}$ **f.** $\sqrt{\dfrac{(8-10)^2+(12-10)^2}{2}}$

5.2 Is $\sqrt{3^2+4^2} = 3+4$? Justify your response.

5.3 Calculate the standard deviation of each of the following sets of numbers.

a. 3, 5, 8, 2, 4, 7, 1, 8, 6, 6

b. 45, 47, 41, 42, 48, 43

c. 19.2, 83.6, 34.5, 74.3, 99.4, 23.3, 52.4, 48.3

5.4 A biology class received the following test scores: 90, 73, 67, 87, 79, 70, 100, 50, 84, 84, 69, 97, 74, 81, 45, 83, 96, 65, 70, and 75. Find the number of students that are within 1 standard deviation of the mean. What percentage of the class do these students represent?

Periodic Assessment—for use after Activity 5

1. Evaluate each of the following expressions.

a. $|-7|$

b. $4|r - s|$, if $r - s > 0$

c. $|-x|$, if $x > 0$

d. $\sqrt{49}$

e. $\sqrt{\frac{3}{8}}$

f. $\sqrt{\frac{(3-6)^2 + (9-6)^2}{2}}$

2. Calculate the mean absolute deviation for the following set of numbers: 46, 36, 65, 53, 40.

3. Calculate the standard deviation for the following set of numbers: 3, 8, 6, 5, 9, 7, 4.

Level 2: Volume 1—Who Gets What and Why?

Additional Review—for use with Activity 1

1.1. Find the sum of each set of numbers given in Parts **a** and **b** below. Then express each number as a fraction of the sum.

a. 7, 5, 10, 3

b. 15, 36, 9

1.2. Calculate each of the following:

a. 3/4 of 280

b. 1/6 of 120,000

c. $(7/20) \bullet 20{,}000$

d. $(4/25) \bullet 7500$

1.3. McCoyle County has been ravaged by a spring flood. The table below shows the population of the five towns in the county.

Town	Population
A	1102
B	157
C	61
D	280
E	398

a. Round each town's population to the nearest tens.

b. Use the rounded values from Part **a** to estimate the total population of McCoyle County.

c. Express the population of each town as a fraction of the total.

d. Find the sum of the population fractions from Part **c**.

e. Use the information in Part **c** to allocate $110,000 of flood-relief funds among the five towns.

1.4. An entrepreneur has sold her company for $4,800,000. She plans to share half the proceeds with her employees, based on years of service. The following table shows the service records for each worker. Calculate the amount each employee should receive.

Employee	Years of Service
A	20
B	10
C	24
D	6

Additional Review—for use with Activity 2

2.1. Round each of the following numbers down to the previous integer.

a. 3.11 **b.** 2.87 **c.** –4.23

2.2. Round each number below to the nearest integer.

a. 2.45 **b.** 4.68 **c.** –0.37

2.3. Determine the standard divisor in each of the following situations.

a. a population of 15,672 with a total of 7 representatives

b. a population of 512,715with a total of 161 representatives

2.4. The citizens of Maltby County are represented by 15 county commissioners. The following table shows the population of the six towns in the county.

Town	Population
A	570
B	1505
C	1451
D	1005
E	837
F	532

a. Determine the standard divisor for this population.

b. Calculate the standard quota (to the nearest hundredth) for each town.

c. Use the Jefferson method to apportion the 15 county commissioners among the six towns, modifying the standard divisor if necessary.

2.5. The residents of Cahill are represented by 12 city councilors. The table below shows the population of each neighborhood in Cahill. Use the Webster method to apportion the 12 councilors among the four neighborhoods.

Neighborhood	Population
A	22,354
B	13,496
C	6728
D	5086

2.6. The following table shows the population of the four cities in Joquin County. Use the Hamilton method to apportion 25 representatives among the four cities.

City	Population
A	13,976
B	15,981
C	58,568
D	86,684

Periodic Assessment 1—for use after Activity 2

1. Find the sum of each set of numbers given in Parts **a** and **b** below. Then express each number as a fraction of the sum.

a. 6, 8, 9, 1 **b.** 12, 3, 5

2. Calculate each of the following:

a. 2/5 of 485 **b.** 2/7 of 35,924

3. **a**. Round 7.87 down to the previous integer.

b. Round 3.47 to the nearest integer.

4. Determine the standard divisor for apportioning 101 representatives among a population of 873,192.

5. To prevent overgrazing, a rancher maintains her herd of cattle on three different pastures. Each spring she counts the animals, then assigns a fraction of the herd to each pasture, based on area. Use one of the methods of apportionment you have learned to divide the herd among the three pastures. The following table shows the area of each pasture.

Pasture	**Area (km^2)**
A	160
B	40
C	50

Additional Review—for use with Activity 3

3.1. Determine the arithmetic mean of each of the following pairs of numbers.

a. 4, 9 **b.** 1, 16 **c.** 6, 7

3.2. Determine the geometric mean of each pair of numbers below.

a. 4, 9 **b.** 1, 16 **c.** 6, 7

3.3. Reduce each of the following expressions to a simple fraction.

a. $\dfrac{\frac{2}{5}}{\frac{7}{8}}$ **b.** $\dfrac{\frac{4}{5}}{\frac{2}{15}}$ **c.** $\dfrac{\frac{2}{5}+\frac{3}{4}}{\frac{7}{9}}$

d. $\dfrac{\frac{1}{2}-\frac{2}{5}}{\frac{3}{2}}$ **e.** $\dfrac{\frac{4}{5}+\frac{1}{4}}{\frac{3}{4}}$ **f.** $\dfrac{\frac{5}{4}-\frac{3}{x+1}}{\frac{3}{x+1}}$

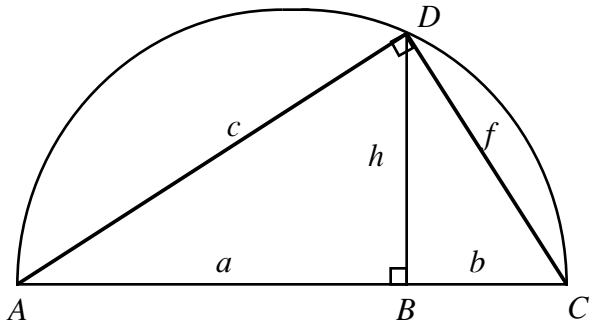
3.4. Evaluate each of the following expressions, rounding your answers to the nearest tenth.

a. $\dfrac{500}{\sqrt{1(1+1)}}$ **b.** $\dfrac{834{,}561}{\sqrt{4(4+1)}}$ **c.** $\dfrac{7904}{\sqrt{3(3+1)}}$

3.5 Evaluate the expression below, given that $M = 945{,}370$; $m = 3$; $W = 713{,}500$; and $w = 2$. Round your answer to the nearest thousandth.

$$\dfrac{\frac{W}{w}-\frac{M}{m+1}}{\frac{M}{m+1}}$$

3.6 Right triangle *ADC* is circumscribed in the semicircle below. If $a = 7$ cm and $b = 3$ cm, what is the value of *h*?



Additional Review—for use with Activity 4

4.1. Determine the priority number in each of the following situations.

a. a population of 3,609,311 with 10 proposed representatives

b. a population of 1,937,088 with 6 proposed representatives

4.2. The table below shows the results of the most recent census for McCoyle County. Use the Huntington method to apportion 29 representatives among the five towns.

Town	Population
A	2105
B	500
C	294
D	760
E	1037

Periodic Assessment 2—for use after Activity 4

1. Determine both the arithmetic mean and geometric mean for each of the following pairs of numbers. Round your answers to the nearest hundredth.

a. 4, 16 **b.** 10, 11

2. Reduce each of the following expressions to a simple fraction.

a. $\dfrac{\frac{3}{7}}{\frac{15}{14}}$ **b.** $\dfrac{\frac{4}{9}+\frac{5}{6}}{\frac{3}{2}}$ **c.** $\dfrac{\frac{7}{x}-\frac{5}{x}}{\frac{3}{x}}$

3. Evaluate each of the following expressions. Round your answers to the nearest tenth.

a. $\dfrac{17{,}603}{\sqrt{2(2+1)}}$ **b.** $\dfrac{1{,}341{,}685}{\sqrt{16 \bullet 17}}$

4. Determine the priority number for a population of 81,457 with 17 proposed representatives.

Level 2: Volume 1
What Are My Child's Chances?

Additional Review—for use with Activity 1

1.1 List the sample space for each of the following experiments:

a. flipping two coins

b. rolling two ordinary six-sided dice

c. playing "Rock, Paper, Scissors" with two people

1.2 Describe the complement of each event below.

a. A fair coin lands heads up.

b. A green light appears on a traffic signal.

c. A test is scored an "A."

1.3 If the probability of an event is $1/6$, what is the probability of its complement?

1.4 If the probability of an event is x, what is the probability of its complement?

1.5 A bag contains three kinds of candy: chocolates, caramels, and fruit chews. Each student in a class of 24 chooses one piece of candy from the bag. Fifteen students take chocolates, while 8 get caramels. The remaining students select fruit chews. Determine each of the following experimental probabilities:

a. P(choosing a chocolate)

b. P(not choosing a caramel)

c. P(choosing a fruit chew)

d. P(not choosing a chocolate)

Additional Review—for use with Activity 2

2.1 Consider an experiment that involves flipping two fair coins.

a. Determine the theoretical probability of obtaining one head and one tail.

b. If you were to flip two coins 1200 times, how many times would you expect to obtain tails on both coins?

2.2 The Burkett family has 4 children—3 girls and 1 boy. The children's father claims that a family is more likely to have 3 children of one gender and 1 of the other than they are to have 2 of each. Assuming that the probability of having a girl or a boy is the same, is he correct? Use theoretical probabilities to support your response.

2.3 A quiz has 10 true-or-false questions. If a student selects answers at random, what is the probability of getting all the answers correct?

2.4 When rolling two ordinary six-sided dice, which sum is the least likely outcome? Justify your response.

2.5 In the game of blackjack, the dealer places one card face down and one face up for herself, while each player receives two cards face down. If the dealer receives an ace, along with a face card (jack, queen, king) or a ten, then she wins automatically. Suppose that the dealer has an ace showing, while the lone player has no face cards or tens in his hand. What is the theoretical probability that the dealer will win automatically?

Periodic Assessment 1—for use after Activity 2

1. The Pineville Cultural Center conducted a poll of entertainment preferences. In this poll, each person could vote for only one event. The results appear below.

Type of Entertainment	Number of Votes
jazz concert	230
classical music concert	299
opera	69
Broadway musical	391
Shakespeare play	276
ballet	92

a. How many people were polled?

b. What is the experimental probability that a person voted for a jazz concert?

c. What is the probability that a person did not vote for a ballet?

2. A laboratory rat shows a recessive trait. One of its parents shows the same trait, while the other does not. Draw the Punnett square for this cross and determine the theoretical probability that another offspring of the same parents has the genotype XX.

3. Imagine that your mother has placed slips of paper labeled with various household chores in a coffee can. There are 9 slips in all, 3 involving yard work, 3 involving housework, and 3 involving meal preparation. Each day you must draw three slips at random, then perform these chores.

a. List the sample space of different types of chores you can draw each day.

b. What is the theoretical probability that your chores will consist of 1 yard work and 2 houseworks?

c. What is the complement of the event described in Part **b**? What is the complement's probability?

Additional Review—for use with Activity 3

3.1 Identify each pair of events below as independent or dependent.

a. Event A: Attending Jefferson High School

Event B: Playing on the Jefferson High soccer team

b. Event A: Going to the movies

Event B: Eating breakfast

c. Event A: Driving a car

Event B: Wearing jeans

3.2 Suppose that A and B are independent events. Calculate $P(A \text{ and } B)$ in each of the following situations:

a. $P(A) = 1/3$ and $P(B) = 1/2$

b. $P(A) = 0.125$ and $P(B) = 0.625$

c. $P(A) = 5/6$ and $P(B) = 6/7$

3.3 Determine the theoretical probability of each event below.

a. A fair coin lands tails up and an ace is drawn from a standard deck of playing cards.

b. A pair of ordinary six-sided dice are rolled twice. On each roll, the numbers on the two dice match.

c. A family with two children has a girl for the first child and a boy for the second child. (Assume that the probability of having a girl or a boy is the same.)

3.4 In 1999, 43,114 people were awaiting kidney transplants (according to the U.S. Institute of Diabetes and Digestive and Kidney Diseases). Of these people, 8523 received kidneys from cadavers and 3922 from living donors. Of patients who receive a kidney from a cadaver, 78.93% survive for 5 years after the transplant. Of those who receive a kidney from a living donor, 90.21% survive for five years after the transplant.

a. Based on this data, what is the probability that a person awaiting transplant will receive a kidney from a cadaver? What is the probability of receiving a transplant from a living donor?

b. What is the probability that a person awaiting transplant will receive a kidney from a cadaver and survive for five more years? What is the probability of receiving a kidney from a living donor and surviving for five more years?

Additional Review—for use with Activities 4 and 5

4.1 Identify each pair of events below as mutually exclusive or not mutually exclusive.

a. A: being a father B: being female

b. A: born in the U.S.A. B: born to run

c. A: a veteran of World War II B: less than 50 years old

d. A: resident of California B: resident of Montana

4.2 Suppose that A and B are independent events and not mutually exclusive. Calculate $P(A \text{ or } B)$ in each of the following situations:

a. $P(A) = 2/3,\ P(B) = 1/4$

b. $P(A) = 0.42,\ P(B) = 0.7$

c. $P(A) = 3/5,\ P(B) = 1/2$

4.3 To protect animal populations, many states limit the number of permits available to hunt certain species. In Montana, these special permits are distributed using a lottery system. Each drawing is conducted independently and receiving a permit for one species does not affect the chances of receiving a permit for another species. The table below shows the numbers of residents and nonresidents who applied for special permits for the 1999 season in two different hunting districts.

District	Species	Permits Available	Resident Applicants	Nonresident Applicants
380-01	elk	100	2864	165
	antelope	50	142	5
	moose	5	208	1
380-02	elk	725	1255	36
	antelope	50	58	1
	moose	2	58	0

Source: Montana Department of Fish, Wildlife & Parks.

a. Determine the probability that a person who applied for all three species in district 380–02 received all three permits.

b. Determine the probability that a person who applied for elk and moose permits in district 380–01 received both permits.

c. Determine the probability that a person who applied for elk and antelope permits in district 380–02 received one permit or the other.

Periodic Assessment 2—for use after Activity 5

1. Describe an example of each of the following:

a. two independent events

b. two dependent events

c. two mutually exclusive events

d. two events that are not mutually exclusive

2. Suppose that A and B are independent events and not mutually exclusive. Calculate $P(A \text{ and } B)$ and $P(A \text{ or } B)$ for each of the following:

a. A: drawing a spade from a standard deck of playing cards

B: rolling a sum of 7 with a pair of ordinary six-sided dice

b. A: having a recessive trait (x) with parents whose genotypes are Xx and xx

B: having a dominant trait (Y) with parents whose genotypes are Yy and Yy

c. A: having a family with three children in the order boy–girl–girl

B: winning a raffle by buying 1 of the 50 tickets sold

3. The table below shows some data (in millions) for the 1998 U.S. Congressional elections.

Age (in years)	Population	Married	Registered to Vote	Voted
18–24	25.5	4.0	9.8	NA
25–34	38.6	22.9	20.2	10.8
35–44	44.4	31.5	27.7	18.1
45–64	57.4	41.5	40.8	30.8
65 +	32.3	18.1	24.4	19.2

Source: U.S. Bureau of the Census.

a. Determine the probability that a person older than 65 was married and registered to vote in 1998.

b. Determine the probability that a person younger than 25 was married or registered to vote in 1998.

c. Determine the probability that a person from 25–44 years old was married or not registered to vote in 1998.

Level 2: Volume 2—There's No Place Like Home

Additional Review—for use with Activity 1

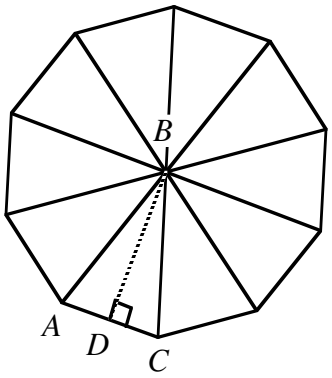
1.1 Find the measure of a central angle of each of the following regular polygons:

a. dodecagon

b. 24-gon

c. nonagon

1.2 The diagram below shows a decagon with a radius of 12 m.



Calculate each of the following:

a. the measures of $\angle ABC$ and $\angle ABD$

b. the length of $\overline{AD}$

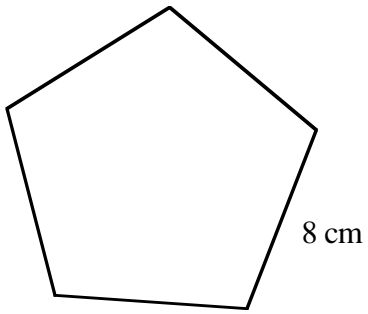
c. the length of $\overline{AC}$

d. the length of $\overline{BD}$

e. the area of ΔABC

f. the area of the decagon

1.3 Find the area of the regular pentagon below. Show your work.



Additional Review—for use with Activity 2

2.1 A regular octagonal prism has a height of 24 m. The bases have a side length of 8 m.

a. Draw a sketch of the prism, labeling the known lengths.

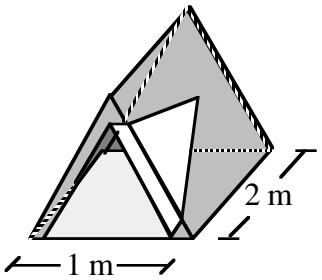
b. Sketch a net for the prism, labeling the known lengths.

c. Determine its lateral area.

d. Calculate the area of the bases.

e. Determine the total surface area of the prism.

2.2 A tent manufacturer is designing a one-person camping tent. The front and back of the tent will be equilateral triangles that measure 1 m on each side, and the side walls will be 2 m long. The tent also will have a floor.



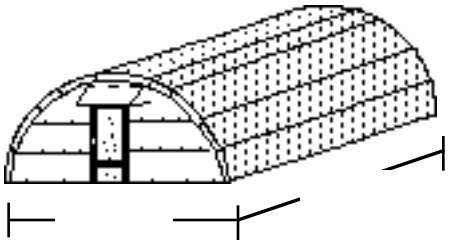
a. Sketch a pattern that could be used to create the tent.

b. Determine the amount of fabric necessary to construct the tent.

c. Calculate the area of the tent floor.

d. Determine the tent's amount of head room (height from floor to peak).

2.3 Donna wants to build a greenhouse for her flower business. The greenhouse must have at least $25\ m^2$ of floor space. The end walls must be large enough to accommodate a door 1 m wide, with shelves 1.5 m wide along each side.



a. Label the diagram with measurements that meet Donna's requirements.

b. The end walls will be covered with plywood. If a sheet of plywood measures 1.25 m by 2.5 m, how many should Donna buy

c. The rest of the greenhouse will be covered in clear plastic, which is sold in rolls 2.5 m by 30 m. How many rolls of plastic should Donna buy?

Additional Review—for use with Activity 3

3.1 Determine the volume of each figure described below:

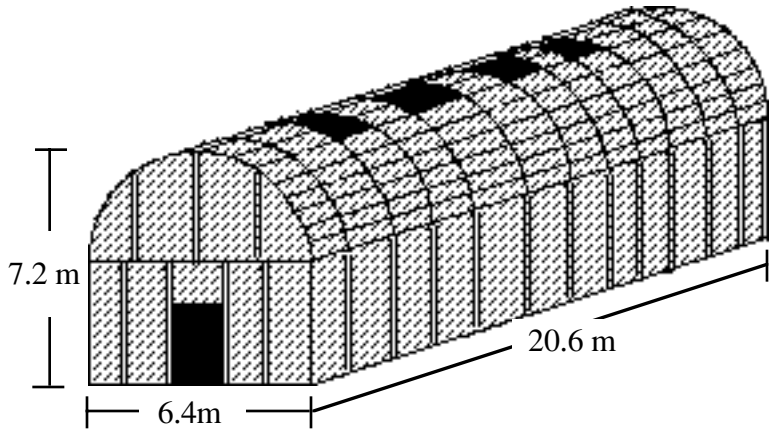
a. a right octagonal prism with height 13 cm and base area 138 cm^2

b. a right cylinder with base radius 4.3 dm and height 27.9 dm

c. a right prism with height 54 cm whose base is an irregular polygon with area 312 cm^2

3.2 Find the volume of a hexagonal prism 24 cm high, whose bases have an apothem of 12 cm.

3.3 The building shown in the diagram below is shaped like a rectangular prism covered by half a cylinder.



a. What is the surface area of the building's four rectangular walls?

b. What is the surface area of the roof?

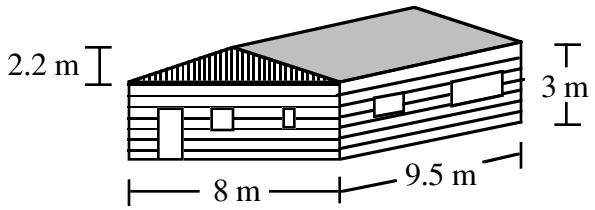
c. What is the total surface area of the building, excluding the floor?

d. What is the total volume of the building?

Periodic Assessment 1—for use after Activity 3

1. Find the surface area and volume of a right prism 34 cm high, whose base is a regular decagon with side length 16.8 cm.

2. Sharone's construction company has just completed the frame of the house shown in the diagram below.



a. Sharone plans to cover the roof with shingles. One package of shingles will cover approximately $3.1\ m^2$. How many packages should she buy?

b. The rest of the house will be painted. One liter of paint covers approximately $6\ m^2$. How many liters should Sharone buy?

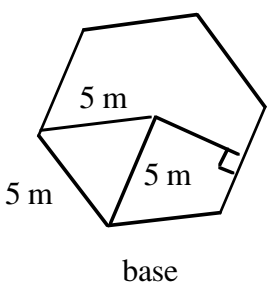
c. The living space of a home is the area of the floor. What is the living space in this house?

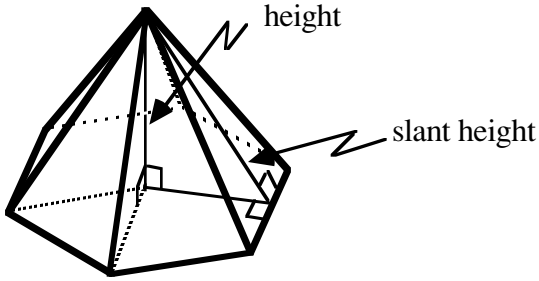
d. The space between the ceiling and the roof is the attic. Sharone wants to install a ventilation fan in the attic. The size of the fan depends on the attic's volume. Determine the volume of the attic.

e. The size of the furnace required depends on the volume of space to be heated. Determine the volume of the house, excluding the attic.

Additional Review—for use with Activity 4

4.1 The diagram below shows a right regular hexagonal pyramid and its base.





a. Sketch a net for the pyramid.

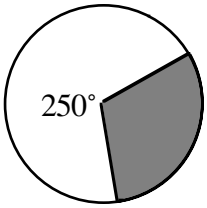
b. Find the length of the apothem of the base and the area of the base.

c. Determine the slant height of the pyramid.

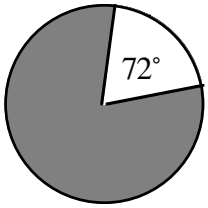
d. What is the lateral surface area of the pyramid? What is the total surface area?

4.2 Each circle shown in Parts **a–d** below has a radius of 6 cm. Find the area of each unshaded sector, then determine the length of the arc that bounds it. (The measures of the central angles of the unshaded sectors are shown.)

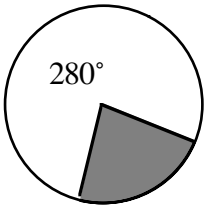
a.



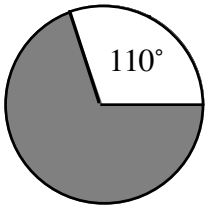
b.



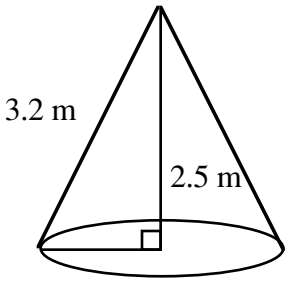
c.



d.



4.3 The diagram below shows a cone.



a. Find the radius and area of the base.

b. What is the lateral surface area of the cone? (The formula for the lateral surface area of a cone is πrs.) What is the total surface area?

Additional Review—for use with Activity 5

5.1 Determine the volume of each of the following figures:

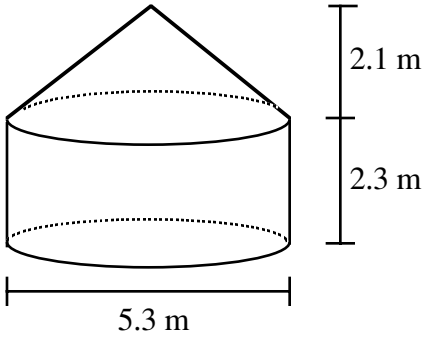
a. a right circular cone with radius 8 cm and height 12 cm

b. a sphere with diameter 8 dm

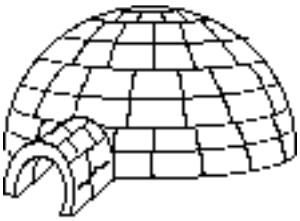
c. a right circular cone with diameter 16 cm and slant height 10 cm

5.2 Find the surface area and volume of a sphere with radius 4.8 m.

5.3 The figure in the following diagram is shaped like a cylinder topped by a cone.



5.4 The main room of the igloo shown in the diagram below is shaped like a hemisphere. Its walls are made from blocks of snow 0.2 m thick. Its outside diameter is 7 m.



a. Find the volume of the hemisphere formed by the igloo's outside diameter.

b. Find the volume of the hemisphere formed by the igloo's inside diameter.

c. What is the volume of the snow used to make the igloo?

d. The entryway of the igloo is shaped like half a cylinder, with an inside radius of 2.2 m and a length of 3 m. What is the interior volume of the entryway?

Periodic Assessment 2—for use after Activity 5

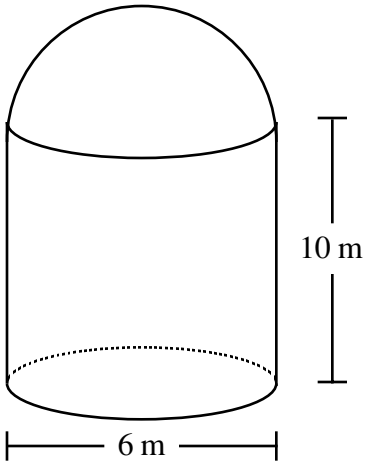
1. Find the surface area and volume of each of the following:

a. a sphere with diameter 8.4 cm

b. a cone with height 12 m and slant height 13 m

c. a square pyramid with slant height 41 cm and a base with side length 18 cm

2. The Dahl family recently built a silo to store grain on their cattle ranch. The silo's dimensions are shown below.



a. The grain capacity is measured in 100-L units called hectoliters (hL). Assuming that $1\ m^3 = 10\ hL$, determine how much grain the Dahls could store in the silo.

b. The Dahls want to paint the silo. If 1 L of paint covers approximately $6\ m^2$, how much paint should they buy?

Level 2: Volume 2—Making Concessions

Additional Review—for use with Activity 1

1.1 Write an inequality to describe each of the following statements.

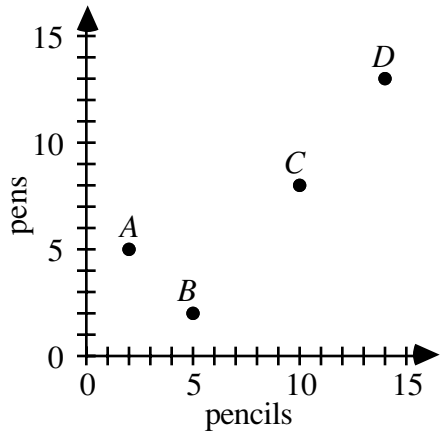
a. The value of n is greater than 5.

b. The value of k is less than or equal to 1.

c. The value of x is less than 2 but greater than –4.

d. The value of r is less than or equal to –5 but greater than –14.

1.2 Describe what each labeled point in the graph below might represent.



1.3 Graph each of the following systems of inequalities, shading the feasible region, and labeling the corner points.

a. $\begin{cases} x \geq 3 \\ x < 7 \\ y \geq 0 \\ y \leq 4 \end{cases}$

b. $\begin{cases} x \geq 4 \\ y \geq -2 \\ x + y \leq 10 \\ 2x + y \leq 18 \end{cases}$

1.4 At G&S Jewelry, two jewelers create rings and necklaces. Together, they work a total of 80 hr per week. It takes approximately 1.5 hr to make a ring and about 1 hr to make a necklace. Each ring typically contains about 5 g of metal, while each necklace contains about 20 g. The store owner limits the amount of metal the store can purchase each week to 500 g. The expected profit from the sale of a ring is $40, while the profit from the sale of each necklace is $70. What should the jewelers do to help maximize the store's profit? Justify your response.

Additional Review—for use with Activity 2

2.1 Identify which of the following lines, if any, are parallel.

a. $3x + 4y = 12$ **b.** $2x + 6y = -9$ **c.** $y = \frac{3}{4}x + 6$

d. $y = -\frac{3}{4}x - 10$ **e.** $3x = 4y - 2$ **f.** $2x + 6y = 35$

2.2 Solve each of the following systems of equations.

a. $\begin{cases} y = 3x - 1 \\ y = -5x + 7 \end{cases}$ **b.** $\begin{cases} 4x - 3y = 11 \\ 6x - y = 13 \end{cases}$ **c.** $\begin{cases} x - y = 3 \\ -\frac{1}{3}x + \frac{1}{3}y = 1 \end{cases}$

d. $\begin{cases} 2x + y = 6 \\ 3x = 4y + 12 \end{cases}$ **e.** $\begin{cases} 3x + y - 3 = 0 \\ 2x - 3y - 2 = 0 \end{cases}$ **f.** $\begin{cases} 2x + 5y = -20 \\ x = -\frac{5}{2}y - 10 \end{cases}$

2.3 Graph the following lines on a single coordinate system, labeling all points of intersection.

$$\begin{cases} x = 0 \\ y = x + 5 \\ 4x + 3y = 36 \\ 2x - y = 8 \\ y = 0 \end{cases}$$

2.4 Given that $P = 6x - 10y$, find the value of P for each of the following:

a. $(0,6)$ **b.** $(-3,5)$ **c.** $(4,-1)$ **d.** $(-10,0)$

2.5 Graph the feasible region described by the following constraints, labeling each corner point. Then determine the maximum value of the objective function $P = 5y - 2x$.

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 3x + 2y \leq 12 \\ 2x + y \leq 7 \end{cases}$$

Periodic Assessment 1—for use after Activity 2

1. Plot and label each of the following on an appropriate coordinate system.

a. the point that represents 5 girls and 3 boys

b. the point that represents 2 girls and 7 boys

2. Graph the system of inequalities below, shading the feasible region, and labeling the corner points.

$$\begin{cases} x \geq -2 \\ y \geq 3 \\ x + y \leq 12 \\ y \leq x + 8 \end{cases}$$

3. Solve each of the following systems of equations.

a. $\begin{cases} x + 5y = -9 \\ 4x - 3y = -13 \end{cases}$ **b.** $\begin{cases} -7x + y = 2 \\ 28x - 4y = 2 \end{cases}$

4. Given that $P = 4x + 3y$, determine the value of P for each of the following:

a. $(5, -2)$ **b.** $(-3, -10)$

Additional Review—for use with Activity 3

3.1 In Parts **a–f** below, perform the indicated operation, if possible, on the given matrices.

a. $\begin{bmatrix} 3 & -2 & 14 \\ -7 & 0 & 4 \end{bmatrix} + \begin{bmatrix} -40 & 3 & 12 \\ -2 & 5 & 12 \end{bmatrix}$

b. $\begin{bmatrix} 5 & 0 \\ -9 & 6 \\ 12 & 5 \end{bmatrix} - \begin{bmatrix} 4 & 10 \\ -7 & 15 \\ 3 & 9 \end{bmatrix}$

c. $\begin{bmatrix} 5 & 7 & 2 \\ -1 & 2 & 9 \\ 8 & 15 & 10 \end{bmatrix} + \begin{bmatrix} -4 & 11 \\ 16 & 3 \\ 6 & 14 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 8 & -3 & 6 \\ -4 & 2 & -3 & 1 \end{bmatrix} \bullet \begin{bmatrix} 7 \\ 5 \\ -3 \\ 6 \end{bmatrix}$

e. $\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} \bullet \begin{bmatrix} -6 & 8 \\ -3 & 2 \\ 5 & 5 \end{bmatrix}$

f. $6 \bullet \begin{bmatrix} 12 & -6 \\ 4 & -10 \end{bmatrix}$

3.2 Find the inverse of each of the following matrices and show how you checked your solutions.

a. $\begin{bmatrix} 3 & -2 \\ -2 & 5 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 7 & -1 \\ 3 & 2 & -5 \end{bmatrix}$

c. $\begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$

3.3 Use matrices to solve each system of equations below.

a. $\begin{cases} 0.1x + 0.2y = 0.7 \\ 0.01x - 0.01y = 0.04 \end{cases}$

b. $\begin{cases} 4x = 7y - 6 \\ 12x = -9y + 12 \end{cases}$

c. $\begin{cases} x + y + z = 4 \\ 3x - 2y + z = 6 \\ 2x + 5y + 3z = 11 \end{cases}$

d. $\begin{cases} 2x + y + z = 2 \\ 4x - 2y + z = 6 \\ -2x - y - 3z = -6 \end{cases}$

3.4 Jayce has two summer jobs: bagging groceries at the local supermarket, and mowing lawns for his neighbors. The supermarket has offered him a minimum of 20 hr per week, but no more than 30 hr per week. The lawn-mowing business supplies a maximum of 35 hr per week. Jayce has decided to work no more than 50 hr per work. If the supermarket pays $5.50 per hour and mowing lawns pays $6.00 per hour, what schedule should Jayce adopt in order to maximize his earnings?

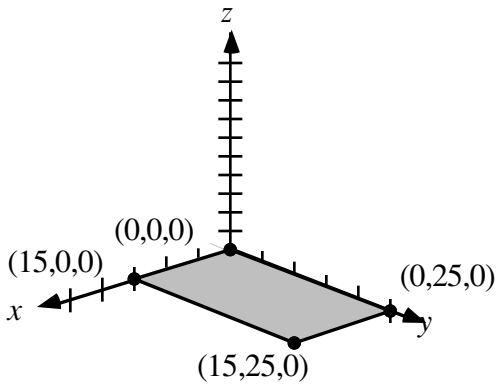
Additional Review—for use with Activity 4

4.1 Plot each of the following points on a three-dimensional coordinate system.

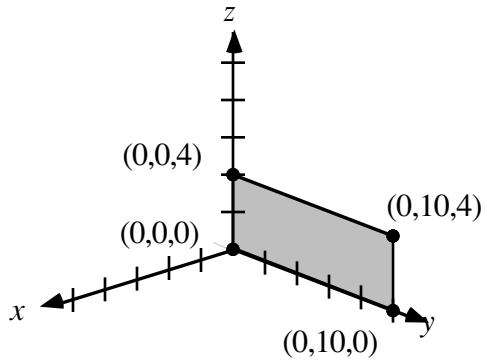
a. $(3,5,1)$ **b.** $(-2,3,4)$ **c.** $(5,-3,-2)$

4.2 Write a system of inequalities to describe each shaded region below.

a.



b.



4.3 Use matrices to solve each of the following systems of equations.

a. $\begin{cases} 2x - 5y + z = -10 \\ x + 2y + 3z = 26 \\ -3x - 4y + 2z = 5 \end{cases}$

b. $\begin{cases} 3x - 2y + z = 1 \\ x - y - z = 2 \\ 6x - 4y + 2z = 3 \end{cases}$

c. $\begin{cases} x + 2y + 3z = 5 \\ -4x + z = 6 \\ 3x - y = -3 \end{cases}$

d. $\begin{cases} x + 2y + z = -0.4 \\ 2x + 2y - z = 3.5 \\ -x - 2y + z = -1.8 \end{cases}$

4.4 In their last basketball game, the Tigers scored 81 points. The number of two-point shots was 1 more than twice the number of free throws, while the number of three-point shots was 3 less than the number of free throws. [Each free throw is worth one point.] How many three-point shots, two-point shots and free throws did they make during the game?

Periodic Assessment 2—for use after Activity 4

1. Find the inverse of each of the following matrices and describe how you checked your solutions.

a. $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 0 & 3 & 4 \end{bmatrix}$

2. Use matrices to solve each of the following systems of equations.

a. $\begin{cases} 4x = y + 6 \\ 2x + 3y = 10 \end{cases}$

b. $\begin{cases} 2x + 4y + 8z = 14 \\ 4x - 2y + 2z = 6 \\ -5x + 3y - z = -4 \end{cases}$

3. Graph the region described by each of the following systems of inequalities on a three-dimensional coordinate system.

a. $\begin{cases} 0 \le x \le 5 \\ 0 \le y \le 3 \\ z = 0 \end{cases}$

b. $\begin{cases} 0 \le x \le -3 \\ 0 \le y \le 4 \\ 0 \le z \le 5 \end{cases}$

4. At the end of their annual fund-raiser, the Key Club had collected 118 bills of three different denominations: ones, fives, and tens. The number of five-dollar bills was 14 less than the sum of the ones and the tens, and there were half as many ones as fives. How much money did they raise?

Level 2: Volume 2—Crazy Cartoons

Additional Review—for use with Activity 1

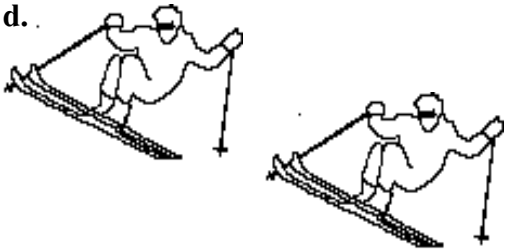
1.1 In Parts **a–d** below, the skier on the left is the pre-image. Identify each transformation as a dilation, translation, rotation, or reflection.

a.

b.

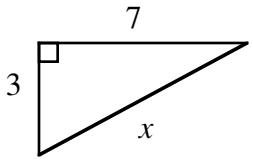
c.

d.

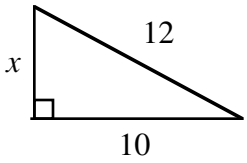


1.2 Determine the value of x in each of the following triangles.

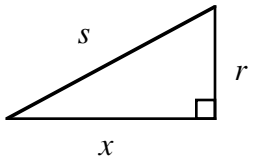
a.



b.



c.



1.3 Determine the distance between each pair of points below.

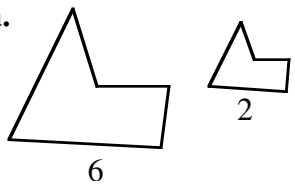
a. (9,14) and (3,6) **b.** (–2,0) and (0,5) **c.** (r,s) and (t,v)

1.4 The ordered pairs $J(4,5)$, $U(4,–3)$, $M(–2,–3)$, and $P(–2,5)$ are the four vertices of a square. List the ordered pairs of the vertices if the square *JUMP* is dilated by each of the following scale factors.

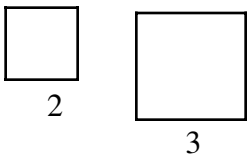
a. 3 **b.** –2 **c.** 3/2

1.5 In Parts **a–c** below, the figure on the left is the pre-image in a dilation. Identify the scale factor of each dilation.

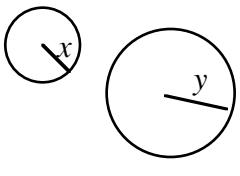
a.



b.



c.



1.6 Find the ratio of the areas of each pair of similar figures in Problem **1.5**.

1.7 Find the ratio of the perimeters of each pair of similar figures in Problem **1.5**.

Additional Review—for use with Activity 2

2.1 **a.** The ordered pairs $A(2,3)$, $B(-3,-1)$, and $C(0,-4)$ are the three vertices of a triangle. Write a matrix **G** to represent these vertices.

b. Use scalar multiplication to write a matrix expression that describes a dilation of the triangle by a scale factor of 4, with center at the origin.

2.2 Write a dilation matrix for each scale factor below.

a. 3 **b.** $-1/4$ **c.** n

2.3 In Parts **a** and **b** below, perform the indicated operation on the given matrices.

a. $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \bullet \begin{bmatrix} -1 & -2 & -3 & 0 \\ 3 & 1 & 0 & -4 \end{bmatrix}$

b. $2 \bullet \begin{bmatrix} -1 & -2 & -3 & 0 \\ 3 & 1 & 0 & -4 \end{bmatrix}$

c. Explain why the results from Parts **a** and **b** are the same.

2.4 The Big Bike store stocks bicycles in four different sizes: 80 cm, 100 cm, 120 cm, and 140 cm. The following matrix represents the Big Bike inventory of mountain and touring bikes.

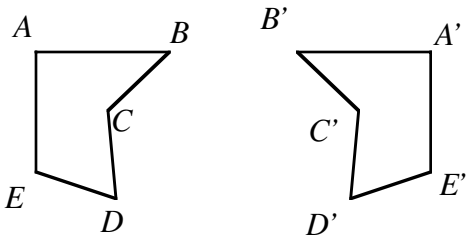
$$\begin{array}{c} \\ \text{mountain} \\ \text{touring} \end{array} \begin{array}{c} \begin{array}{cccc} 80 & 100 & 120 & 140 \end{array} \\ \begin{bmatrix} 3 & 7 & 3 & 4 \\ 5 & 8 & 2 & 1 \end{bmatrix} \end{array}$$

The store plans to triple this inventory for the summer season. Use matrix operations to determine the summer inventory.

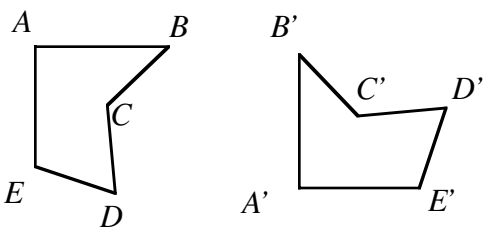
Periodic Assessment—for use after Activity 2

1. In Parts **a–d** below, pentagon *ABCDE* is the pre-image. Identify each transformation as a dilation, translation, rotation, or reflection.

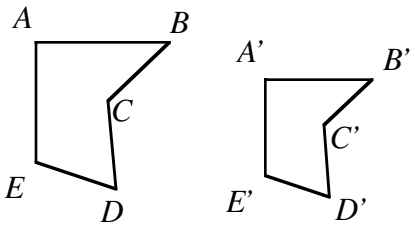
a.



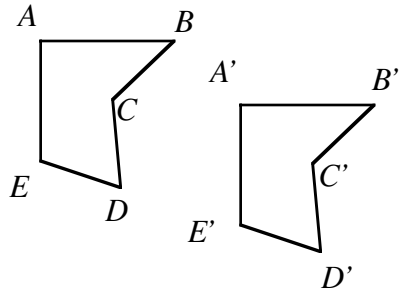
b.



c.

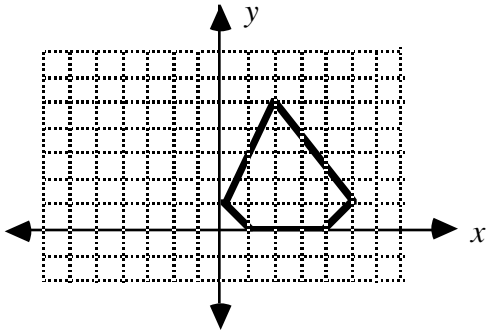


d.



2. Determine the distance between the points (7,–4) and (9,16).

3. **a.** Write a matrix **V** to represent the vertices of the polygon below. (On the grid shown, each square represents 1 unit.)



b. Use scalar multiplication to write a matrix expression that describes a dilation of the polygon by a scale factor of 2, with center at the origin.

4. Write a dilation matrix for a scale factor of $-1/5$.

5. Simplify the following matrix expression:

$$\begin{bmatrix} 4/5 & 0 \\ 0 & 4/5 \end{bmatrix} \bullet \begin{bmatrix} 5 & 10 & -30 & 0 \\ 15 & 10 & 0 & -40 \end{bmatrix}$$

Additional Review—for use with Activity 3

3.1 What must be true about the dimensions of two matrices in order to add them together?

3.2 **a.** What translation matrix would you use to translate a heptagon 3 units to the left and 4 units down?

b. What is the length of the translation vector?

3.3 **a.** What translation matrix would you use to translate a pentagon 7 units to the right and 10 units down?

b. What is the length of the translation vector?

3.4 Matrices **D**, **E**, **F**, **G**, **H**, and **J** below include two matrices used for dilations, two matrices used for translations, and two matrices that represent the vertices of different polygons.

$$\mathbf{D} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} -7 & -5 & 9 & 5 \\ 5 & -2 & -5 & 7 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} 3 & 3 & 3 & 3 \\ -1 & -1 & -1 & -1 \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} -1 & 0 & 1 & 2 & 3 \\ 5 & 0 & 2 & 0 & 5 \end{bmatrix}$$

a. Identify the matrices used for dilation, including their respective scale factors.

b. Identify the translation matrices, including a description of the translation and the length of the translation vector.

3.5 Use the matrices in Problem **3.4** to evaluate each of the following matrix expressions. If an expression cannot be evaluated, explain why not.

a. $\mathbf{D} \bullet \mathbf{E}$ **b.** $\mathbf{G} + \mathbf{H}$ **c.** $\mathbf{E} + \mathbf{F} \bullet \mathbf{J}$ **d.** $\mathbf{H} + \mathbf{D}(\mathbf{G} + \mathbf{H})$

Additional Review—for use with Activity 4

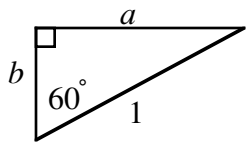
4.1 Identify the letters that have rotational symmetry.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y
Z

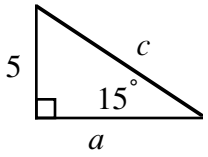
4.2 Which regular polygons have rotational symmetry? Justify your response.

4.3 Determine the unknown lengths in each of the following triangles.

a.



b.



4.4 Matrix **M** below represents the vertices of a triangle.

$$\mathbf{M} = \begin{bmatrix} -5 & -5 & -10 \\ 6 & 3 & 3 \end{bmatrix}$$

In Parts **a–f**, perform the indicated operation and describe the transformation that results.

a. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \bullet \mathbf{M}$ **b.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \bullet \mathbf{M}$ **c.** $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \bullet \mathbf{M}$

d. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \bullet \mathbf{M}$ **e.** $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \bullet \mathbf{M}$ **f.** $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \bullet \mathbf{M}$

4.5 What transformation occurs when you multiply **M** by the following matrix?

$$\begin{bmatrix} t\cos\theta & -t\sin\theta \\ t\sin\theta & t\cos\theta \end{bmatrix}$$

Additional Review—for use with Activity 5

5.1 Identify the letters that have line symmetry.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y
Z

5.2 Which regular polygons have line symmetry? Explain your response.

5.3 Point P has the coordinates (–2, 7). Determine the coordinates of P' after each of the following:

a. a reflection in the x-axis

b. a reflection in the y-axis

c. a reflection in the line $y = x$

d. a reflection in the line $y = -x$

5.4 Matrix **M** below represents the vertices of a triangle.

$$\mathbf{M} = \begin{bmatrix} -5 & -5 & -10 \\ 6 & 3 & 3 \end{bmatrix}$$

In Parts **a–f**, perform the indicated operation and describe the transformation that results.

a. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \bullet \mathbf{M}$ **b.** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \bullet \mathbf{M}$ **c.** $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \bullet \mathbf{M}$

d. $\begin{bmatrix} 4 & 4 & 4 \\ 7 & 7 & 7 \end{bmatrix} + \mathbf{M}$ **e.** $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \bullet \mathbf{M}$ **f.** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \bullet \mathbf{M}$

Periodic Assessment 2—for use after Activity 5

1. **a.** What matrix would translate a triangle 12 units right and 5 units down?

b. What is the length of the translation vector in Part **a**?

2. Use matrices **D**, **E**, **F**, **G**, **H**, and **N** below to simplify the matrix expressions in Parts **a–e**, then describe the resulting transformations.

$$\mathbf{D} = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix} \qquad \mathbf{E} = \begin{bmatrix} 3 & 3 & 3 & 3 \\ -6 & -6 & -6 & -6 \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} 1/3 & 0 \\ 0 & 1/3 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} -7 & -5 & 9 & 5 \\ 5 & -2 & -5 & 7 \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} -1 & 0 & 1 & 2 \\ 5 & 0 & 2 & 0 \end{bmatrix} \qquad \mathbf{N} = \begin{bmatrix} 2 & 2 & 7 \\ -3 & -3 & 4 \end{bmatrix}$$

a. $\mathbf{D} \bullet \mathbf{H}$

b. $\mathbf{G} + \mathbf{E}$

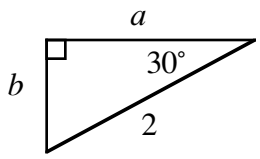
c. $\mathbf{E} + \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \bullet \mathbf{G}$

d. $\mathbf{F}(\mathbf{E} + \mathbf{H})$

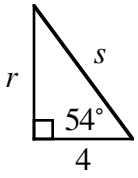
e. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \bullet \mathbf{N}$

3. Determine the unknown lengths in each of the following triangles.

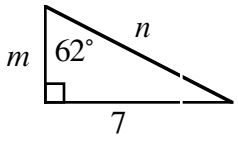
a.



b.



c.

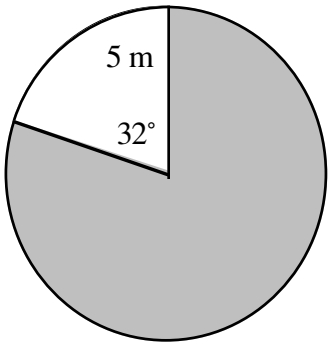


Level 2: Volume 2
Hurry! Hurry! Hurry! Step Right Up

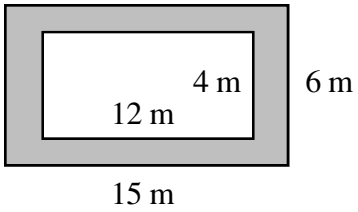
Additional Review—for use with Activity 1

1.1 Parts **a–d** below show four different dart boards. Assuming that a dart has an equal chance of landing on any point on each board, determine the theoretical probability that a dart lands in the shaded region.

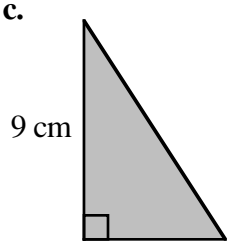
a.



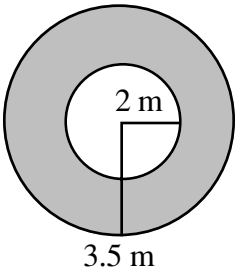
b.



c.



d.



1.2 **a.** Graph all the possible ordered pairs (x, y) where $0 \le x \le 7$, and $0 \le y \le 7$.

b. Given a point (x, y) selected at random from Part **a**, determine the probability that $x + y \le 5$.

Additional Review—for use with Activity 2

2.1 The following table shows the probabilities of the three different outcomes of a given event. Calculate the expected value in this case..

Outcome	Probability
12	1/6
9	1/3
4	1/2

2.2 The table below shows the probabilities of the three different outcomes of another event. Calculate the expected value in this situation.

Outcome	Probability
10	1/6
20	2/3
30	1/6

2.3 The following table shows the prizes for a game of chance. Determine a cost to play that will make this game a fair one.

Prize Value	Probability
$2	1/4
$1	1/2
$2	1/4

2.4 In a coin-drop game, players try to land quarters inside a 6-cm-by-6-cm square on a game board that measures 8.4 cm by 8.4 cm. The game costs $1 to play. If the quarter lands completely within the square, the player wins $5. If any part of the quarter touches or crosses a line on the square, the player loses. **Note:** The diameter of a quarter is approximately 2.4 cm.

a. Make a sketch of the game board. Indicate the area in which the center of the quarter must land to win $5.

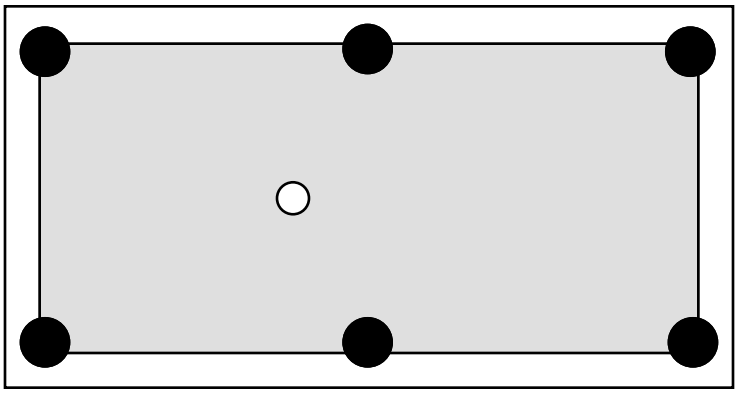
b. What is the probability of winning the game?

c. What is the expected value of this game?

d. Determine a cost to play that will make this game a fair one.

Periodic Assessment 1—for use after Activity 2

1. Sallie's Game Arcade offers a unique promotion for pool players. Before each game, a laser pointer mounted on the ceiling randomly shines a beam of light onto the pool table. If the beam of light shines in one of the pockets, the pool players win a free game. The diameter of each of the four corner pockets is approximately 13.7 cm, while the diameter of each side pocket is approximately 12.4 cm. What is the probability of winning a free game?



1.2 m

2.4 m

2. The following table shows the probabilities of the four different outcomes of an event. Calculate the expected value.

Outcome	Probability
3	1/8
10	3/8
12	3/8
3	1/8

3. What does it mean for a game of chance to be a fair game?

Additional Review—for use with Activity 3

3.1 Describe the differences between dependent events and independent events. Include examples in your explanation.

3.2 What is conditional probability?

3.3 Consider an experiment in which marbles are drawn from a bag, one at a time, with replacement. Determine each of the following probabilities if the bag contains 10 red marbles (R), 8 green marbles (G), and 6 blue marbles (B).

a. $P(\text{B})$

b. $P(\text{R})$

c. $P(\text{GB})$

d. $P(\text{BGR})$

e. $P(\text{RRR})$

3.4 In one carnival game, players draw chips from a box. The box contains 15 chips—5 are green and 10 are yellow. Without looking, each player draws one chip from the container, then without replacing that chip, draws another. If you draw two green chips, you win \$1. If you draw one green and one yellow, you win \$0.50. If you draw two yellow chips, you lose. The game costs \$0.25 to play.

a. Use a tree diagram to show all the different outcomes for this game.

b. What is the probability of drawing each of the following?

1. two green chips

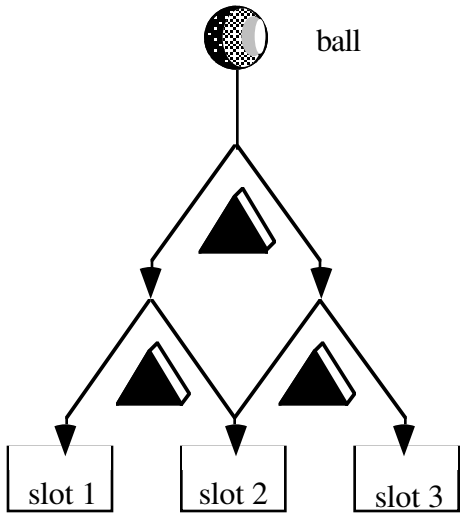
2. one green chip and one yellow chip

3. two yellow chips

c. On average, how much would you expect to win per play?

Periodic Assessment 2—for use after Activity 3

1. A binostat game involves dropping a ball through a triangular grid. At each level of the grid, the ball deflects to the right or left of a peg with equal probability, until it enters a numbered slot. For a ball to reach slot 1 in the two-level binostat shown below, it must deflect left and then left again (LL). To reach slot 2, a ball can deflect left and then right (LR) or it can deflect right and then left (RL). To reach slot 3, the ball must deflect right and then right again (RR).



a. What is the probability of landing in each slot of this game?

b. If it costs $1.50 to play this game, determine some prize values that would make this a fair game.

c. If you place this game twice, are the two games independent events? Justify your response.

2. Draw a diagram to represent a three-level binostat game. Use your diagram to help you complete Parts **a** and **b** below.

a. Complete the following probability distribution table for a three-level binostat game.

Slot	Probability
1	
2	
3	
4	

b. Determine a cost to play and some prize values that would make this a fair game.

Level 2: Volume 2—Atomic Clocks Are Ticking

Additional Review—for use with Activity 1

1.1 Identify the rate of growth or decay represented by each of the following exponential equations:

a. $y = 229 \bullet 0.44^x$ **b.** $y = 750 \bullet 1.10^x$

c. $y = 99.95 \bullet 3^x$ **d.** $y = 82 \bullet 0.7^x$

1.2 Evaluate each equation in Problem **1.1** for the following values: $x = 3$, $x = 0.5$, and $x = 1/3$.

1.3 Evaluate each of the expressions below.

a. $64^{1/2}$ **b.** $16^{1/4}$

c. $15625^{1/6}$ **d.** $\sqrt[3]{343}$

1.4 Write an exponential equation in the form $y = a \bullet b^x$ to model each of the following situations:

a. an initial population of 63 and a growth rate of 14%

b. a decay rate of 0.8% and an initial population of 741

c. an initial population of 2 and a growth rate of 250%

d. an initial population of $4 \bullet 10^9$ and a decay rate of 25%

1.5 From the moment a new car is driven off the lot, its value begins to depreciate. This situation can be modeled using exponential decay. The table below shows how the mean value of a car decreases over time.

Year after Purchase	Mean Value ($)
0	26,756
1	19,700
2	16,738
3	13,937
4	11,775
5	9750

a. Determine the rate of decay in this car's value.

b. Write an equation in the form $y = a \bullet b^x$ to model this situation.

c. Use your equation to estimate the value of the car 10 years after it was purchased.

Additional Review—for use with Activity 2

2.1 Evaluate each of the following expressions:

a. 7^{-2} **b.** $81^{-1/4}$ **c.** $\dfrac{1}{6^{-3}}$

2.2 Rewrite each expression below using positive exponents.

a. $x^{-1/2}$ **b.** $x^{-2/3}$ **c.** $\sqrt[7]{x}$

d. $\dfrac{1}{x^{-9}}$ **e.** $\dfrac{1}{\sqrt[5]{32}}$ **f.** $\dfrac{1}{x^{-3/5}}$

2.3 Simplify each of the following expressions:

a. $(x^5)^2$ **b.** $(x^4)^{-6}$ **c.** $(x^9)^{1/3}$

2.4 **a.** Rewrite $(1/5)^4$ using an integer as the base.

b. Rewrite $(1/x)^y$ using x as the base.

c. Rewrite $(2/7)^5$ without using parentheses.

d. Rewrite $(2/7)^5$ using a negative exponent.

2.5 Solve each of the following equations for x.

a. $x^8 = 390625$ **b.** $x^{1/3} = 7$

c. $x^{3/5} = 2.93$ **d.** $10x^4 = 12960$

2.6 This table shows the change in the temperature of a cup of coffee over time.

Time (min)	Temperature (°C)
0	98
2	94
4	90
6	87
8	85
10	83
12	80
14	78
16	76

a. Create a scatterplot of the data.

b. Find the mean percent decrease in temperature over a 2-min interval, then write an equation of the form $y = a \cdot b^x$ that models the data.

Additional Review—for use with Activity 3

3.1 Find the value of x, to the nearest hundredth, in each of the following equations.

a. $5^x = 100$ **b.** $7^x = 81$ **c.** $12^x = 60$

d. $169^x = 13$ **e.** $64^x = 14$ **f.** $2^x = 0.15$

3.2 Each equation below models the decay of a radioactive substance. Determine the half-life of each substance.

a. $y = 980 \bullet 0.73^x$ **b.** $y = 26 \bullet 0.49^x$

c. $y = 7 \bullet 10^{12} \bullet 0.97^x$ **d.** $y = 1500 \bullet (3/4)^x$

3.3 Determine the value of b in an equation of the form $y = a \bullet b^x$ to model the radioactive decay of a substance with each of the following half-lives.

a. 30 minutes

b. 19 months

c. 115 years

3.4 Due to a merger, a company plans to reduce its work force by a total of 50%. To make the transition easier for both the company and its employees, management plans to eliminate 15% of the existing jobs every three months. If the company currently employs 480 workers, how long will it take to complete the staff reduction?

Periodic Assessment—for use after Activity 3

1. Evaluate each of the exponential expressions below.

a. 2^{-4} **b.** $512^{1/3}$

c. $49^{-1/2}$ **d.** $(3/7)^3$

2. Rewrite each of the following expressions using a single, positive exponent.

a. $(x^7)^9$ **b.** x^{-46} **c.** $\dfrac{1}{x^{-1/4}}$

d. $\dfrac{x^6}{x^2}$ **e.** $\left(\dfrac{4}{x}\right)^{-3}$ **f.** $\left(x^{16}\right)^{1/2}$

3. Identify the rate of growth or decay represented by the equation $y = 375 \bullet (7/11)^x$.

4. **a.** Write an equation in the form $y = a \bullet b^x$ for a population that has a decay rate of –17% and an initial population of 4500.

b. Determine the population after 20 time periods.

c. Find the half-life to the nearest hundredth.

5. An object contains 1 billion radioactive atoms. If they decay at a rate of –22% every 4 yr, how many years will it take for this object to contain less than 1 million radioactive atoms?

Level 2: Volume 3—And the Survey Says . . .

Additional Review—for use with Activity 1

1.1 Describe the differences between each of the following pairs of terms:

a. population and sample

b. parameter and statistic

1.2 Describe any potential sources of bias in each of the following surveys.

a. Miranda is interested in opening a vegetarian burrito shop. Before investing any money, she wants to make sure that people would be interested in such a restaurant. She stood outside the local natural foods store and polled people as they entered.

b. The student council at Richardson High School wants to install bottled-water dispensers in their school. To assess interest in the idea, they randomly selected 20% of the students and polled them.

c. The city council plans to spend some tax revenue on landscaping for the downtown area. Before beginning the project, they want to know if they have community support. Each council member volunteers to call 20 citizens during the lunch hour and report back.

d. Pizza Palace wants to increase its daily sales. The manager is considering adding pasta to the menu. Before making a decision, she asked every sixth customer if he or she would order pasta if it was available.

1.3 A cable television company plans to survey its customers about their opinions on infomercials.

a. What is the population for this poll?

b. Describe one method of sampling the population.

c. What advantages or disadvantages are there to the sampling method you described in Part **c**?

Additional Review—for use with Activity 2

2.1 Identify the type of sampling method used in each of the following situations. Justify your responses.

a. At a battery factory, the finished batteries travel along a conveyor belt to the packaging area. Before they are packaged, an employee removes every 200th battery for testing.

b. At a local bingo parlor, the numbers are written on ping-pong balls and placed in a rotating bin. The caller reaches into the bin and blindly selects one ball at a time.

c. A candidate for mayor divides the city into 48 different districts. She then randomly selects 30 people from each district to visit.

d. A math teacher has a set of index cards on which she has written her students' names. To decide who to call on next, he shuffles the cards and selects one from the set.

2.2 The Transylvania Credit Union plans to install vending machines in the employee cafeteria. The foods offered will depend on a survey of employee preferences. Suggest one way to obtain a sample of 25 employees from this population, using each of the following methods:

a. simple random sampling

b. stratified sampling

c. systematic sampling

2.3 Identify any potential biases in each sampling technique you described in Problem **2.2**.

2.4 **a.** Write an original survey question that shows extreme bias.

b. Rewrite your question from Part **a** so that it shows little or no bias.

Periodic Assessment 1—for use after Activity 2

1. Big Sky University plans to poll their students on whether or not they should reduce hours at the student health service. The administration would like to survey 2000 of their 30,000 students. Suggest a way to obtain a sample using each of the following methods, and describe any potential sources of bias in each one.

a. systematic sampling

b. random sampling

c. stratified sampling

2. How does a statistic differ from a parameter?

3. In each of the following situations, identify the sampling method used and describe any potential sources of bias.

a. For a survey of movie preferences, a company polls the first 30 people that enter a theater.

b. For a survey of student work habits, the student council randomly selects 4 seniors, 4 juniors, 4 sophomores, and 4 freshmen.

c. For a survey of clothing preferences, a manufacturer polls every third person that enters a department store.

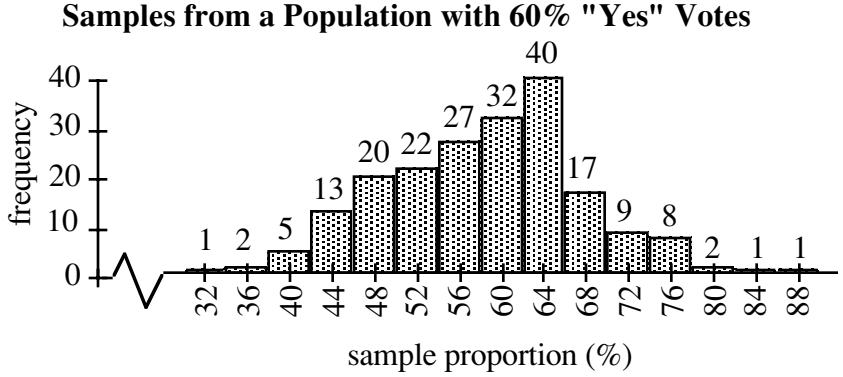
d. For a survey of the track team, the names of all team members are placed in a hat and 9 names are drawn.

e. For a survey of computer usage, a company selects 23 listings from each letter of the alphabet in the telephone directory.

Additional Review—for use with Activity 3

3.1 Some nursing home residents would like to have soft-drink machines installed in each building. According to company policy, 60% of the residents must approve the installation before it can occur. To assess support, the company polls a random sample of 25 residents. Of those surveyed, 12 indicated that they would vote for the soft-drink machines.

To determine what conclusions they could draw from this sample, the company used a computer simulation to collect 200 samples of size 25 from a population in which 60% favored the machines. The following histogram shows the results of this simulation.



a. In the actual company survey, what percentage of the sample indicated that they would vote for the soft-drink machines?

b. The histogram shows the results obtained from 200 simulated samples. What is the mean of the sample proportions from this simulation?

c. Should the company begin contacting vending-machine companies? Justify your response.

3.2 Explain the differences between a population proportion and a sample proportion.

3.3 **a.** Using an ordinary, six-sided die, what is the probability of rolling a two?

b. Use a sampling program to simulate taking 25 samples of 50 rolls each. Sort and list the sample proportions from least to greatest.

c. Create a histogram of the sample proportions and label the axes appropriately.

d. What is the mean of the sample proportions?

e. Based on your simulated data, what is the experimental probability of obtaining fewer than 5 twos in 50 rolls?

f. What is the experimental probability of obtaining more than 10 twos in 50 rolls?

Additional Review—for use with Activity 4

4.1 Calculate the standard deviation in each of the following situations.

a. a population proportion of 55% and a sample size of 81

b. a population proportion of 90% and a sample size of 16

c. a population proportion of 40% and a sample size of 60

4.2 Determine the maximum value of 2 standard deviations for each of the following sample sizes.

a. 100

b. 75

c. 600

4.3 In a random sample of 900 voters, 500 planned to support Candidate A.

a. What was the sample proportion?

b. Determine the maximum value of 2 standard deviations in this situation.

c. Write a confidence statement using the information above, including a margin of error.

4.4 In a sample of 200 batteries, 15 were defective.

a. What percentage of the batteries were defective?

b. What is the margin of error for this sample?

c. Write a confidence statement for this situation.

4.5 A random sample of 4000 college students were asked the following question: "Do you have a job to help pay for your education?" Of those surveyed, 3267 responded "yes." Write a confidence statement that describes the results of this survey.

Periodic Assessment 2—for use after Activity 4

1. **a.** A four-sided die has the numbers 1, 2, 3, and 4 printed on it. If each roll is equally likely to occur, what is the probability of rolling a 3?

b. Use a sampling program to simulate taking 25 samples of 50 rolls each. Sort and list the sample proportions from least to greatest.

c. Create a histogram of the sample proportions and label the axes appropriately.

d. What is the mean of your sample proportions?

e. Based on your simulation, what is the experimental probability of obtaining fewer than five 3s in 50 rolls?

f. What is the experimental probability of obtaining more than ten 3s in 50 rolls?

2. Determine the standard deviation in each of the following situations:

a. a population proportion of 80% and a sample size of 250

b. a population proportion of 30% and a sample size of 120

3. Find the maximum value of 2 standard deviations for each of the following sample sizes:

a. 1000

b. 320

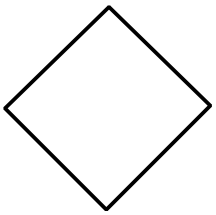
4. A random sample of 213 mall employees is given the following question: "Do you take regular breaks at work?" Of those surveyed, 178 responded "yes." Write a confidence statement that describes the results of the survey, including a margin of error.

Level 2: Volume 3—Traditional Design

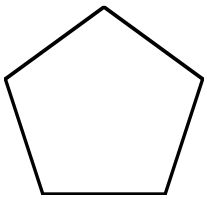
Additional Review—for use with Activity 1

1.1 For each figure below, draw all the lines of symmetry and describe any rotational symmetry.

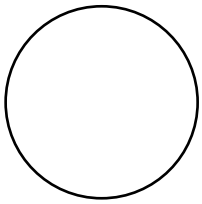
a.



b.



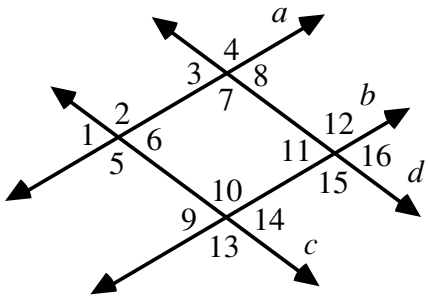
c.



1.2 **a.** If $\Delta SZD \sim \Delta KMP$, what relationship exists between each pair of corresponding angles?

b. If $\Delta SZD \sim \Delta KMP$, what proportion describes the relationship between corresponding sides?

1.3 In the diagram below, lines *a* and *b* are parallel, and lines *c* and *d* are parallel. Use this diagram to identify each set of angles described in Parts **a–d**.



a. all pairs of vertical angles

b. all pairs of corresponding angles

c. all pairs of alternate interior angles

d. all pairs of alternate exterior angles

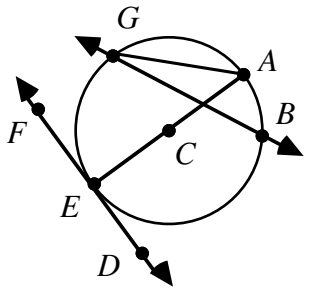
1.4 Use the diagram given in Problem **1.3** to complete the following.

a. Name all angles congruent to $\angle 1$.

b. Name all angles congruent to $\angle 2$.

Additional Review—for use with Activity 2

2.1 Use the diagram below to identify each set described in Parts **a–f**.



a. all chords

b. all secants

c. all tangents

d. all points of tangency

e. all diameters

f. all radii

2.2 Use the diagram in Problem **2.1** to answer the following questions.

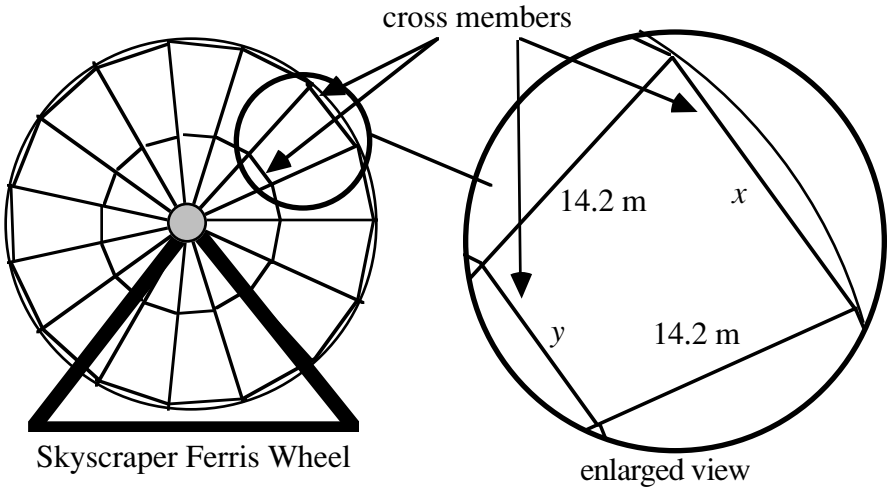
a. Why is $\overleftrightarrow{BG}$ not a chord?

b. Why is $\overline{AG}$ not a radius?

c. Why is $\overleftrightarrow{FD}$ not a secant?

d. Why is $\overline{AG}$ not a secant?

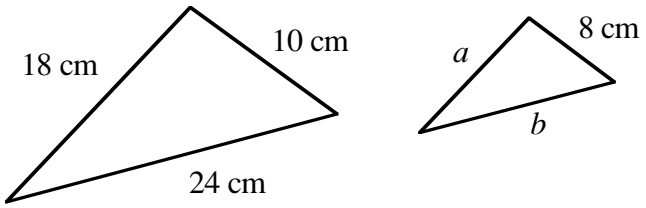
2.3 The Mall of America in Minneapolis, Minnesota, features several carnival rides, including the Skyscraper Ferris Wheel. The wheel is approximately 21.3 m high. It has 15 gondolas that each carry 6 passengers. As shown in the diagram below, the frame requires cross members for strength. Use what you know about the chords of a circle to determine the lengths of cross members x and y. (Hint: Draw a perpendicular from the wheel's center and use trigonometric ratios.)



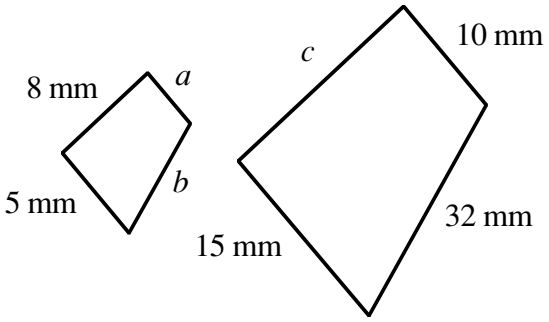
Additional Review—for use with Activity 3

3.1 Determine the unknown measurements in each pair of similar figures below.

a.

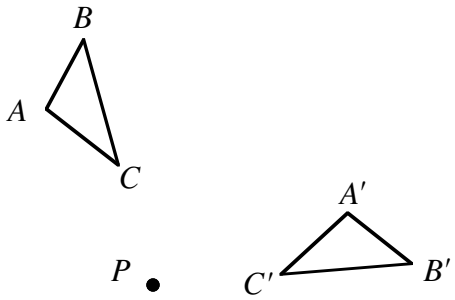


b.

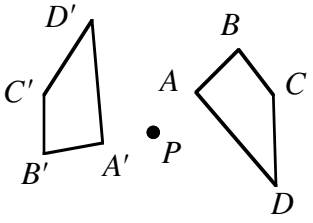


3.2 Determine the angle of rotation for each of the following transformations.

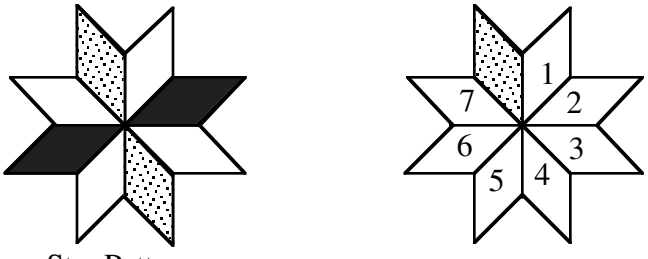
a.



b.



3.3 The design shown below is a star pattern favored by many quilters. The shaded quadrilateral (on the right) is the polygon used to create the pattern.



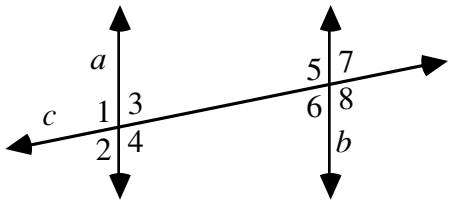
Star Pattern

a. Describe the shaded quadrilateral.

b. Identify a transformation of the shaded quadrilateral that would result in each of the polygons 1 through 7.

Periodic Assessment—for use after Activity 3

1. In the following diagram, lines *a* and *b* are parallel and are cut by transversal *c*. Use this diagram to identify each set described in Parts **a–d**.



a. all pairs of vertical angles **b.** all pairs of corresponding angles

c. all pairs of alternate interior angles **d.** all pairs of alternate exterior angles

2. Use the diagram in Problem **1** to complete the following.

a. Name all angles congruent to $\angle 1$ and state why they are congruent.

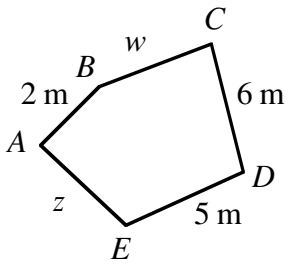
b. Name all angles congruent to $\angle 2$ and state why they are congruent.

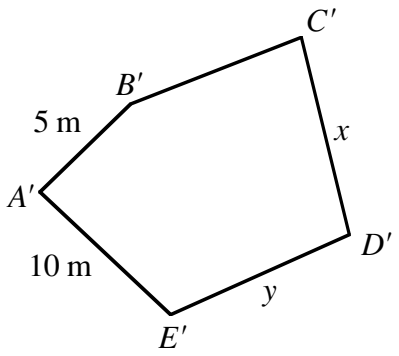
3. Draw and label a circle with the following:

a. radius $\overline{AB}$ **b.** secant $\overleftrightarrow{BC}$ **c.** chord $\overline{BC}$

d. tangent $\overleftrightarrow{CD}$ **e.** point of tangency C **f.** diameter $\overline{EB}$

4. Determine the unknown measurements in the pair of similar figures below.





Level 2: Volume 3—If the Shoe Fits . . .

Additional Review—for use with Activity 1

1.1 Simplify each of the following expressions.

a. $|-0.54|$ **b.** $|1.3|$ **c.** $|0|$

d. $|13 - 9|$ **e.** $|5 - 21|$ **f.** $|5.63 - 13.97|$

1.2 Determine the equation, in slope-intercept form, of the line through the points (5,13) and (9,27).

1.3 Use the equation $y = 7.93x + 1.54$ to complete Parts **a** and **b** below.

a. If $x = 15$, what is y?

b. If $y = 54.12$, what is x?

1.4 Saving money for a college education is an important goal for most families. The table below shows how the costs of college tuition have changed over time, where the cost for the years 1982–84 represents an index of 100.

Year	Cost Index	Predicted Index	Residual	Absolute Value of Residual
1980	70.8			
1985	119.9			
1986	129.6			
1987	139.4			
1988	150.0			
1989	161.9			
1990	175.0			
1991	192.8			
1992	213.5			

Source: *The American Almanac 1993–1994*. City, State: The Reference Press, 1993. p. 484.

a. Create a scatterplot of the data in the table.

b. Find and graph a linear equation that fits the data reasonably well.

c. Use your equation to calculate a predicted index for each year. Then determine the residuals, along with the absolute value of each residual.

d. Find the sum of the absolute value of the residuals. Modify your linear model to lower the sum.

e. Use your best model to predict the index for college tuition in the current year. Describe what this value represents.

Additional Review—for use with Activity 2

2.1 Determine the median for each of the following sets of data.

a. 70, 75, 72, 75, 75, 80, 73

b. 30, 28, 27, 38, 32, 46, 49, 48

c. 95, 90, 83, 65, 76, 68

2.2 In Parts **a–d** below, modify the equation of the given line to correspond with its image following the indicated vertical shift.

a. $y = 2x - 7$; down 3.2 units

b. $y = (2/3)x + 1/2$; up 1/6 unit

c. $y = -2.3x - 1.4$; up one-third of the vertical distance between (2,6) and (3,9)

d. $y = 0.3x - 5$; down one-third of the vertical distance between (3,7) and (–3,–1)

2.3 Describe how you would divide a data set of 16 values into three groups. What about a set of 17 values?

2.4 Explain how to find the median-median line for a data set.

Periodic Assessment—for use after Activity 2

1. **a.** Determine the equation, in slope-intercept form, of the line through the points (1970,195) and (1991,933).

b. Use your equation from Part **a** to determine the value of y when $x = 1986$.

2. The table below shows how the life expectancy of U.S. residents has changed over time.

Year	Life Expectancy (years)	Predicted Life Expectancy	Absolute Value of Residual
1970	70.8		
1975	72.6		
1980	73.7		
1981	74.1		
1982	74.5		
1983	74.6		
1984	74.7		
1985	74.7		
1986	74.7		
1987	74.9		
1988	74.9		
1989	75.1		
1990	75.4		
1991	75.5		
1992	75.8		
1993	75.5		
1994	75.7		
1995	75.8		
1996	76.1		
1997	76.5		

Source: *Statistical Abstract of the United States, 1999*. Washington, DC: U.S. Bureau of the Census, 1999, p. 88.

a. Create a scatterplot of the data, representing the year 1970 as $x = 0$.

b. Determine the equation of the median-median line for this data set.

c. Calculate the sum of the absolute value of the residuals for your model from Part **b**.

d. Use your model to predict the life expectancy of U.S. residents in the years 2000 and 2050.

Additional Review—for use with Activity 3

3.1 Describe a set of data for which you would expect the median-median line and a linear regression equation to differ greatly.

3.2 The table below shows how the acreage of farm land in the United States has changed over time.

Year	Farm Land (millions of acres)	Predicted Farm Land	Absolute Value of Residual
1980	1039		
1985	1012		
1986	1005		
1987	999		
1988	995		
1989	991		
1990	987		
1991	983		
1992	980		
1993	978		
1994	975		

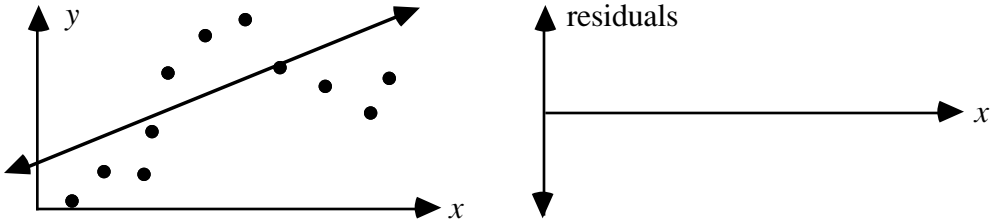
Source: *Statistical Abstract of the United States, 1995*. Washington, DC: U.S. Bureau of the Census, 1995. p. 674.

a. Create a scatterplot of the data, representing the year 1980 as $x = 0$.

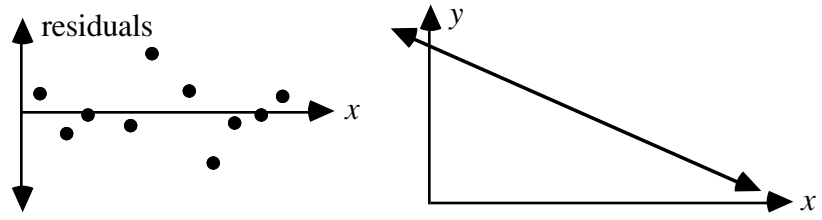
b. Find and graph the equation of the regression line for this data set.

c. Create a residual plot for your model. What does the residual plot indicate about the appropriateness of this model?

3.3 The graph on the left shows a scatterplot and its regression line. Use this information to sketch the corresponding residual plot. Discuss what the residual plot indicates about how well the regression equation models the data.

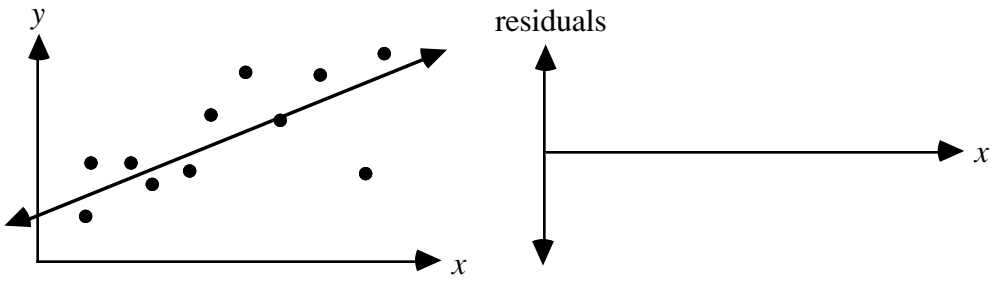


3.4 The graph on the left shows a residual plot. Use this information to sketch the corresponding scatterplot on the graph of the regression line. Discuss what the residual plot indicates about how well the regression equation models the data.

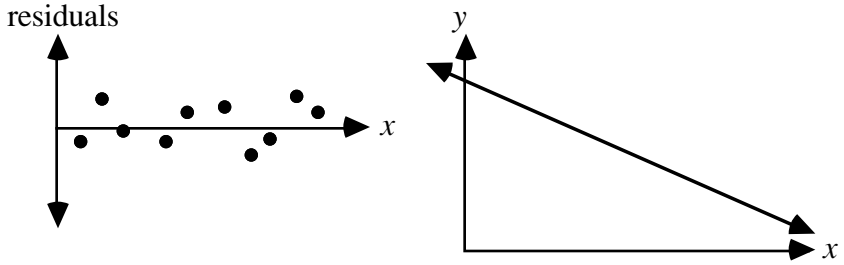


Periodic Assessment 2—for use after Activity 3

1. The graph on the left shows a scatterplot and its regression line. Use this information to sketch the corresponding residual plot. Discuss what the residual plot indicates about how well the regression equation models the data.



2. The graph on the left shows a residual plot. Use this information to sketch the corresponding scatterplot on the graph of the regression line. Discuss what the residual plot indicates about how well the regression equation models the data.



3. The equation of the linear regression for the data set below is $y = 0.9x + 0.33$. The equation of the median-median line for this data is $y = 0.91x + 0.11$. Which line provides a better model for the data? Justify your response.

x	***y***	***x***	***y***
1	2	9	8
2	1	10	9
3	3	11	15
4	6	12	12
5	5	13	10
6	7	14	14
7	5	15	13
8	3	16	15

Level 2: Volume 3—Take It to the Limit

Additional Review—for use with Activity 1

1.1 Write the first five terms of each sequence defined below.

a. $t_n = 15 - 11(n - 1)$

b. $k_n = 1.3 \bullet 3^{n-1}$

1.2 Identify the number of terms in each of the following sequences. (Hint: Determine an explicit formula for the sequence, then solve the formula for the term number *n*.)

a. 12, 6, 0, . . . , –126

b. 5, 15, 45, . . . , 2,657,205

c. 32, 16, 8, . . . , 0.125

d. 7, 10, 13, . . . , 58

1.3 Complete Steps **1–3** for each sequence below.

1. Identify the sequence as either arithmetic or geometric.

2. Write an explicit formula for each sequence.

3. Identify the next two terms in the sequence.

a. 18, 27, 40.5, 60.75, 91.125, . . .

b. 3, 9, 15, 21, 27, . . .

c. 15, 2, –11, –24, –37, . . .

d. –6, 12, –24, 48, –96, . . .

1.4 Titus wants to gain some weight for the upcoming football season. He buys a protein drink which advertises that it will help a person gain 2% of body weight each month. Titus weighs 73 kg.

a. Write an explicit formula to model this situation.

b. Football season begins in 8 months. If the manufacturer's claim is true, what will Titus weigh when football season begins?

1.5 An auditorium has 18 seats in the front row. Each successive row has 2 more seats.

a. Write an explicit formula for the sequence which models the number of seats per row.

b. If the last row of the auditorium has 56 seats, how many rows are there?

Additional Review—for use with Activity 2

2.1 Determine the sum of the first 1000 natural numbers.

2.2 Complete Steps **1–4** for each sequence below.

1. What is the first term of the sequence?

2. What is the common difference?

3. How many terms are in the sequence?

4. What is the sum of the terms of the sequence?

a. 5, 12, 19, . . . , 187

b. 13, 32, 51, . . . , 241

2.3 Determine the sum of each sequence below.

a. 21, 29, 37, 45, . . . , 109

b. 7, 14, 21, 28, . . . , 126

2.4 Blaize wants to recruit new members for the Community Service Club. She hopes to increase the membership by 3 students per week. Every week, she hands out coupons for a local movie theater to each member. The club originally had 21 members. If her plan succeeds, how many coupons will she distribute during the 9-month school year?

2.5 Consider an arithmetic sequence with a first term of 9 and a 53rd term of 191.

a. Determine the sum of the first 53 terms.

b. Identify the common difference of the sequence.

c. List the second and third terms of the sequence.

Periodic Assessment 1—for use after Activity 2

1. Complete Steps **1–4** for each sequence below.

1. What is the first term of the sequence?

2. What is the common difference?

3. How many terms are in the sequence?

4. What is the sum of the terms of the sequence?

a. 3, 12, 21, . . . , 291

b. 21, 36, 51, . . . , 801

2. An auditorium has 22 seats in the front row. Each successive row has 4 more seats.

a. Write an explicit formula for the sequence that models the number of seats per row.

b. If the last row in the auditorium has 234 seats, how many rows are there?

c. How many seats are there in the entire auditorium?

Additional Review—for use with Activity 3

3.1 The formula for finding the sum of a geometric series is shown below.

$$S_n = \frac{g_1 r^n - g_1}{r - 1}$$

a. What does g_1 represent?

b. What does r represent?

c. What does n represent?

d. Use the formula to find the sum of a geometric series with 10 terms whose first term is 5 and whose common ratio is a.

3.2 Determine the sum of the terms of each of the following:

a. a geometric sequence in which the first term is 5, the common ratio is 3, and the number of terms is 25

b. a geometric sequence in which $g_1 = 2.4$, $r = 0.9$, and $n = 12$

c. $4 + 8 + 16 + \cdots + 262{,}144$

d. $40 + 32 + 25.6 + \cdots + 2.74877906944$

3.3 Identify each of the following series as arithmetic or geometric and determine its sum.

a. $4 + 7 + 10 + \cdots + 76$

b. $8 + 32 + 128 + \cdots + 157{,}464$

c. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{1024}$

3.4 Kavita let go of her helium balloon. It rose for 6 min before popping. In the first minute, it rose 40 m. In each successive minute, the distance it rose decreased by a factor of 0.6.

a. Write an explicit formula that models this situation.

b. Write the corresponding series for the first six terms of the geometric sequence.

c. How high did the balloon rise before it popped?

Additional Review—for use with Activity 4

4.1 Identify the infinite sequences below that appear to have a limit, explaining your reasoning in each case. If the sequence has a limit, determine the sum of the sequence.

a. 512, 602, 692, 782, 872, . . .

b. 400, 200, 100, 50, 25, . . .

c. 14, 3, –8, –19, –30, . . .

d. 6, 18, 54, 162, 486, . . .

e. 15, 4.5, 1.35, 0.405, 0.1215, . . .

4.2 Consider the following infinite geometric sequence:

$$2, \frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \ldots$$

a. What is the common ratio?

b. What does its limit appear to be?

c. After how many terms of the sequence are all successive terms within 0.001 of the limit?

4.3 Consider an infinite geometric sequence for which $g_1 = 0.42$ and $r = 0.5$.

a. List the first three terms of the sequence.

b. What is the sum of the infinite geometric series formed by this sequence?

4.4 Nathan told his math teacher that he was going to buy a pair of jeans after school. He announced that since he had four 25%-off coupons, he was going to get the jeans for free. His teacher tried to explain that—even if he was allowed to use all four coupons—he still would not get the jeans for free. The jeans originally cost $38.

a. Write a geometric sequence to model the cost of the jeans if Nathan used 0, 1, 2, 3, or 4 coupons.

b. If Nathan was able to use an infinite number of coupons, would he ever get the jeans for free? Explain your response.

Periodic Assessment 2—for use after Activity 4

1. Find the sum, if possible, of each geometric series below. If it is not possible, explain why not.

a. $2.5 + 7.5 + 22.5 + \cdots + 3{,}985{,}807.5$

b. $12 + 18 + 27 + 40.5 + 60.75 + \cdots$

c. $100 + 80 + 64 + 51.2 + 40.96 + \cdots$

d. a series in which the first term is 9, the common ratio is 2, and the number of terms is 14

2. Consider the following infinite geometric sequence:

$$9,\ 3,\ 1,\ \frac{1}{3},\ \frac{1}{9},\ \ldots$$

a. What is the common ratio?

b. What does the limit appear to be?

c. After how many terms of the sequence are all successive terms within 0.001 of the limit?

d. What is the sum of the series formed by this sequence?

Level 2: Volume 3—Algorithmic Thinking

Additional Review—for use with Activity 1

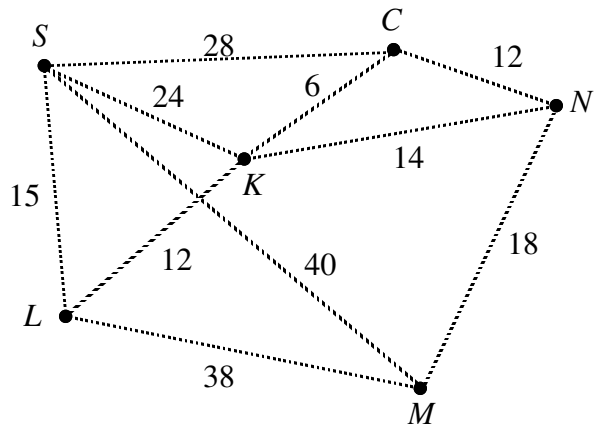
1.1 List the first three prime numbers.

1.2 List all the prime numbers between 50 and 100.

1.3 **a.** Write an algorithm for approximating the square root of a number to the nearest tenth, without using the square-root button on a calculator.

b. Identify any decision points in your algorithm from Part **a**.

1.4 The diagram below shows six bus stations and the distances between them in kilometers. Use the nearest neighbor algorithm to identify the shortest route that visits each station, beginning and ending at stop *L*.



Hint: The nearest neighbor algorithm consists of the following steps:

- Starting with any vertex, draw an edge to its nearest vertex.
- Continue this process from the second vertex, drawing an edge to the next nearest vertex not yet visited and so on, until all vertices have been visited.
- To complete a Hamiltonian circuit, return to the original vertex.

1.5 Write an algorithm that could be used to solve each of the following for x:

a. $4x + 7 = 19$

b. $-2(x + 3) \leq -8$

Additional Review—for use with Activity 2

2.1 Draw a flowchart for finding the absolute deviation of a given set of data based on the following steps.

Step 1: Calculate the mean of the data.

Step 2: Find the difference between each data value and the mean.

Step 3: Square each difference from Step 2.

Step 4: Find the mean of the squared values from Step 3.

2.2 Draw a flowchart for subtracting any two-digit number from another two-digit number of greater value, based on the following steps.

Step 1: Input a pair of two-digit numbers.

Step 2: If the ones digit in the larger number is greater than or equal to the ones digit in the smaller number, proceed to Step 5. If not, continue to the next step.

Step 3: Reduce the tens digit in the larger number by 1.

Step 4: Add 10 to the ones digit in the larger number.

Step 5: Find the difference between the value in the ones digit for the larger number and the ones digit for the smaller number.

Step 6: Record the difference from Step 5.

Step 7: Find the difference between the tens digit for the larger number and the tens digit for the smaller number.

Step 8: Record the difference from Step 7.

Step 9: Add the recorded values from Steps 6 and 8.

Step 10: Output the value from Step 9 as the answer.

2.3 Design and draw a flowchart to help teach a young child to cross a busy intersection that features a signal with the messages WALK and DON'T WALK.

Additional Review—for use with Activity 3

3.1 **a.** Identify each of the following sequences as arithmetic, geometric, or neither.

1. 4, 8, 16, 32, 64, 128, …

2. 4, 7, 10, 13, 16, 19, …

3. 1, 1, 2, 3, 5, 8, …

4. 243, 81, 27, 9, 3, …

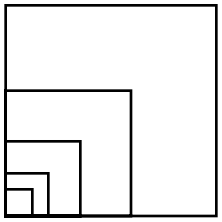
b. Write a recursive formula for each sequence in Part **a**.

3.2 **a.** Use each algorithm below to generate the first 4 terms of a sequence.

1. $\begin{cases} a_n = 15 \\ a_n = a_{n-1} + (-7) \end{cases}$ **2.** $\begin{cases} g_n = 2 \\ g_n = 5g_{n-1} \end{cases}$

b. Which steps in the formulas in Part **a** creates recursion?

3.3 The diagram below shows a sequence in which each square is a dilation by a scale factor of 0.6 of the next larger square. Write an algorithm that could be used to recreate this diagram.



Periodic Assessment—for use after Activity 3

1. List all the prime numbers between 30 and 50.

2. Write an algorithm that could be used to solve the equation $5 + -2x = 19$.

3. Draw a flowchart for calculating the geometric mean of two random integers based on the following steps.

Step 1: Randomly pick an integer.

Step 2: If the integer is positive, continue with the next step. If the integer is negative or 0, repeat Step 1.

Step 3: Randomly pick another integer.

Step 4: If integer is different from the one in Step 1, continue with the next step. If the integer is the same, repeat Step 3.

Step 5: If the integer is positive, continue with the next step. If the integer is negative or 0, repeat Step 3.

Step 6: Find the product of the two integers.

Step 7: Calculate the square root of the product from Step 6.

4. Identify each of the following sequences as geometric or arithmetic and write a recursive formula for it.

a. $-2,\ -5,\ -8,\ -11,\ -14,\ \ldots$

b. $-2,\ 8,\ -32,\ 128,\ -512,\ 2024,\ \ldots$

5. Use the following algorithm to generate the first 4 terms of a sequence:

$$\begin{cases} g_n = 400 \\ g_n = \frac{1}{4} g_{n-1} \end{cases}$$